International Committee on
Global Navigation Satellite Systems

# GLONASS
# Reference Frame Evolution

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FOURTEENTH MEETING OF THE INTERNATIONAL COMMITTEE ON
GLOBAL NAVIGATION SATELLITE SYSTEMS

ICG-14

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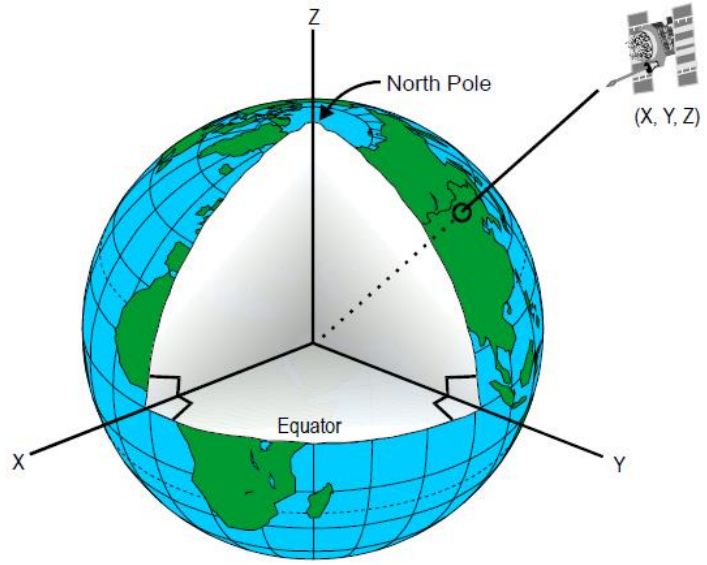
# System of Geodetic Parameters PZ-90

The **«Parametry Zemli 1990»** (**PZ-90**) (*in English* **«The Earth Parameters 1990»**) is a System of the Earth Geodetic Parameters including:

- Fundamental Geodetic Constants ($fM = 398600{,}4418\ m^3s^{-2}$; $\omega = 7{,}292115\times10^{-5}\ s^{-1}$)
- Reference Ellipsoid Parameters ($a = 6378136{,}0$ m; $\alpha = 1/298{,}25784$)
- Earth's Gravity Field Model ($n = m = 360$)
- **Terrestrial Reference System** (PZ-90.11)
- **Transformation Parameters** (with ITRF2008, ITRF2014, WGS 84)

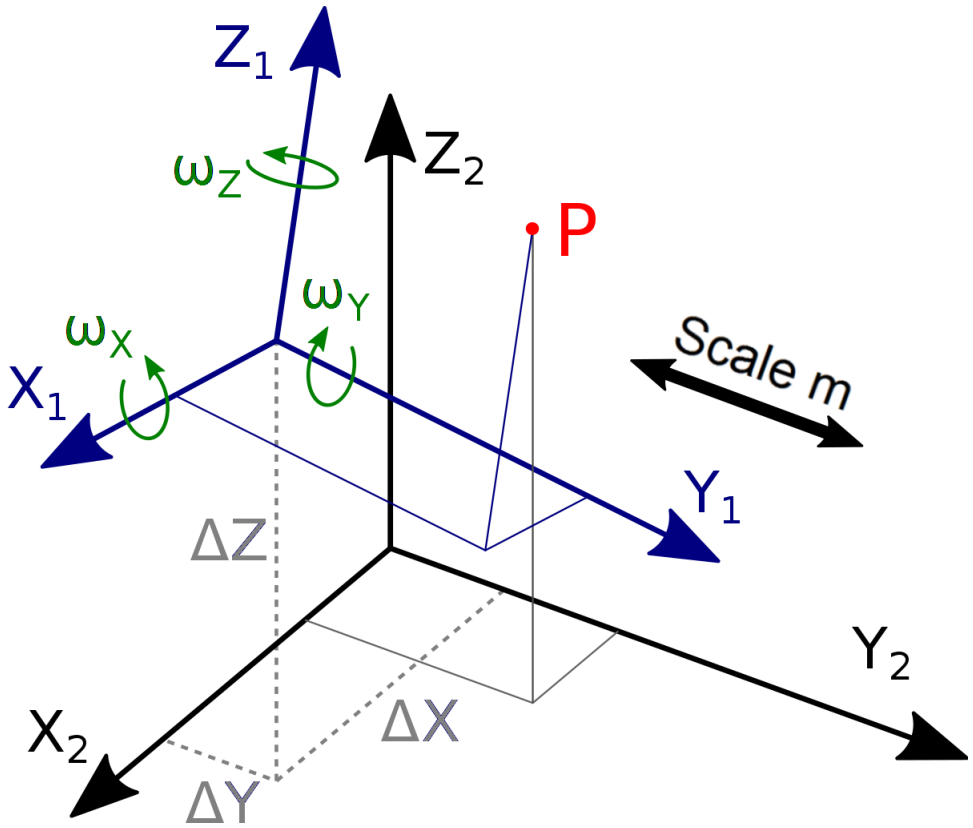
The System of the Earth Geodetic Parameters PZ-90 is developed by the Military Topographic Department of the General Staff of Armed Forces




## PZ-90 Reference Frame Realizations

| # | TRF Version | Date of Issue | Use in GLONASS | | Epoch | Precision of the Relative Station Position | Absolute Accuracy |
|---|---|---|---|---|---|---|---|
| | | | Start | End | | | |
| 1 | PZ-90 | 1990 | 1993 | Sep 2007 | NA | 30-50 cm | 3-10 m |
| 2 | PZ-90.02 | 2002 | Sep 2007 | Jan 2014 | 2002,0 | 2-3 cm | 30-50 cm |
| 3 | PZ-90.11 | 2011 | Jan 2014 | Up to Now | 2011,0 | 5-10 mm | 5 cm |

# Helmert Transformation




Standard Transformation between two Reference Systems is a Helmert Transformation

$$\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}_P = (1+m)\begin{pmatrix} 1 & \omega_Z & -\omega_Y \\ -\omega_Z & 1 & \omega_X \\ \omega_Y & -\omega_X & 1 \end{pmatrix}\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}_P + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}$$

consists of 7 parameters:

$\Delta X, \Delta Y, \Delta Z$ are 3 translation parameters
specified the TRF-origin

$\omega_X, \omega_Y, \omega_Z$ are 3 rotation angles
specified the TRF-orientation

$m$ is a scale parameter

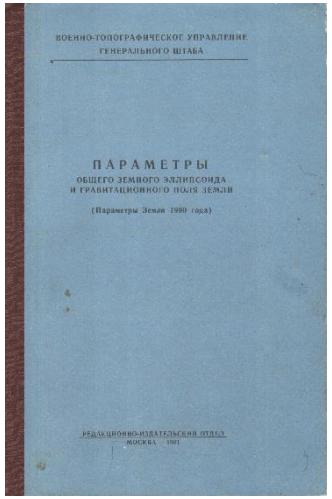
# Transformation Parameters at Epoch 2010.0 from PZ-90.11 to ITRF2014

| # | From | To | $\Delta X$ (m) | $\Delta Y$ (m) | $\Delta Z$ (m) | $\omega_X$ (mas) | $\omega_Y$ (mas) | $\omega_Z$ (mas) | $m$ ($10^{-6}$) | Epoch |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | PZ-90 | PZ-90.02 | –1.07 ±0.10 | –0.03 ±0.10 | +0.02 ±0.10 | 0 | 0 | –130 ±10 | –0.220 ±0.020 | 2002.0 |
| 2 | WGS 84 (G1150) | PZ-90.02 | +0.36 ±0.10 | –0.08 ±0.10 | –0.18 ±0.10 | 0 | 0 | 0 | 0 | 2002.0 |
| 3 | PZ-90.11 | ITRF2008 | –0.003 ±0.002 | –0.001 ±0.002 | +0.000 ±0.002 | +0.019 ±0.072 | –0.042 ±0.073 | +0.002 ±0.090 | –0.000 ±0.0003 | 2010.0 |
| 4 | PZ-90.11 | ITRF2014 | –0.0053 ±0.0020 | –0.0040 ±0.0020 | –0.0032 ±0.0020 | +0.035 ±0.073 | –0.087 ±0.073 | +0.036 ±0.090 | –0.0000 ±0.0001 | 2010.0 |

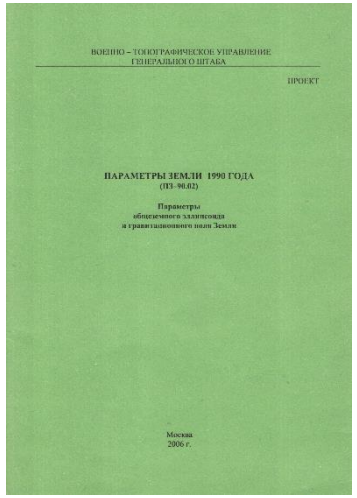
was presented on previous ICG-13 meeting in Xi'an, China

$$\mathrm{RMS}_{\vec{X}^{\mathrm{PZ-90.11}}_{\mathrm{ITRF\,2014}}} = 1,2\ \mathrm{cm}$$

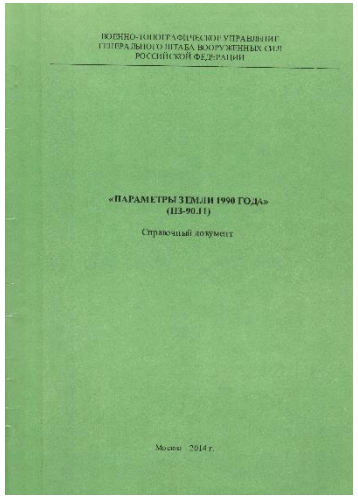
PZ-90 short description (PZ-90 template) was already updated and available at
http://www.unoosa.org/documents/pdf/icg/2019/resources/PZ-90.11_v.1.2_04.11.2018.pdf




PZ-90



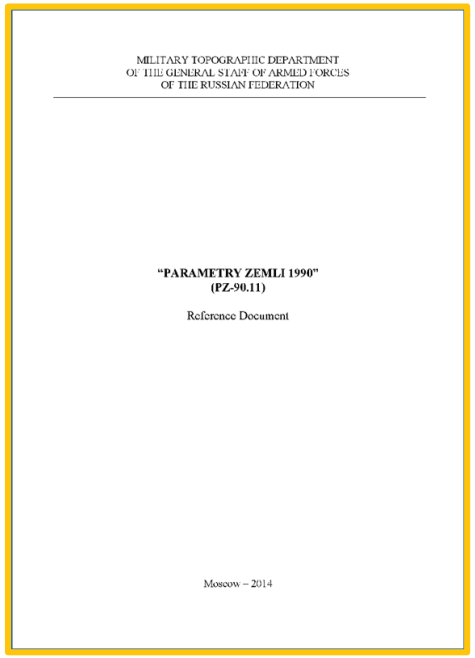

PZ-90.02




PZ-90.11

**Updated Reference Document on PZ-90 is preparing to issue**

Current English Version of Reference Document on PZ-90 available at
http://eng.mil.ru/files/PZ-90.11_final-v8.pdf
First Presented at ICG-10 in 2015, Boulder, USA


MILITARY TOPOGRAPHIC DEPARTMENT
OF THE GENERAL STAFF OF ARMED FORCES
OF THE RUSSIAN FEDERATION

"PARAMETRY ZEMLI 1990"
(PZ-90.11)

Reference Document

Moscow – 2014

# Methods for Determination of the Transformation Parameters


1) Direct using the same station coordinates in both reference frames was performed to calculate transformation parameters at Epoch 2010.0 from PZ-90.11 to ITRF2014.
36 IGS Sites were used to calculate transformation parameters from PZ-90.11 to ITRF2014.


**2) Using of GNSS orbits that were determined from ground stations specified in two reference frames.**

# PZ-90.11 Transformation Parameters Determination through GLONASS Orbits

55° 54' 44" N, 37° 48' 29" E
ИАЦ INFORMATION AND ANALYSIS CENTER FOR POSITIONING, NAVIGATION AND TIMING

https://www.glonass-iac.ru/en/

Data Span: 07/2005 – 10/2019
Solution Type: Daily
$x_i = (\Delta X, \Delta Y, \Delta Z, \omega_X, \omega_Y, \omega_Z \text{ or } m)_i$.
Mean value was averaged over year

$$\overline{x} = \frac{\sum x_i}{n}, \text{ generally } n = 365$$

except 2005 ($n$ = 172) and 2019 ($n$ = 172).
Standard Deviation (StD) is

$$StD = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}}.$$

Units for all 7 parameters are **cm**.
Rotation angles and Scale were converted at the Mean Earth Radius
$R_e$ = 6371000,0 m

1 milli arc second (mas) = $10^{-3}$ arc second
1 mas = 3,09 cm at $R_e$

1 part per billion (ppb) = $10^{-9}$
1 ppb = 0,64 cm at $R_e$

The Root of the Sum of Square is

$$RSS_7 = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 + \omega_X^2 + \omega_Y^2 + \omega_Z^2 + m^2}$$

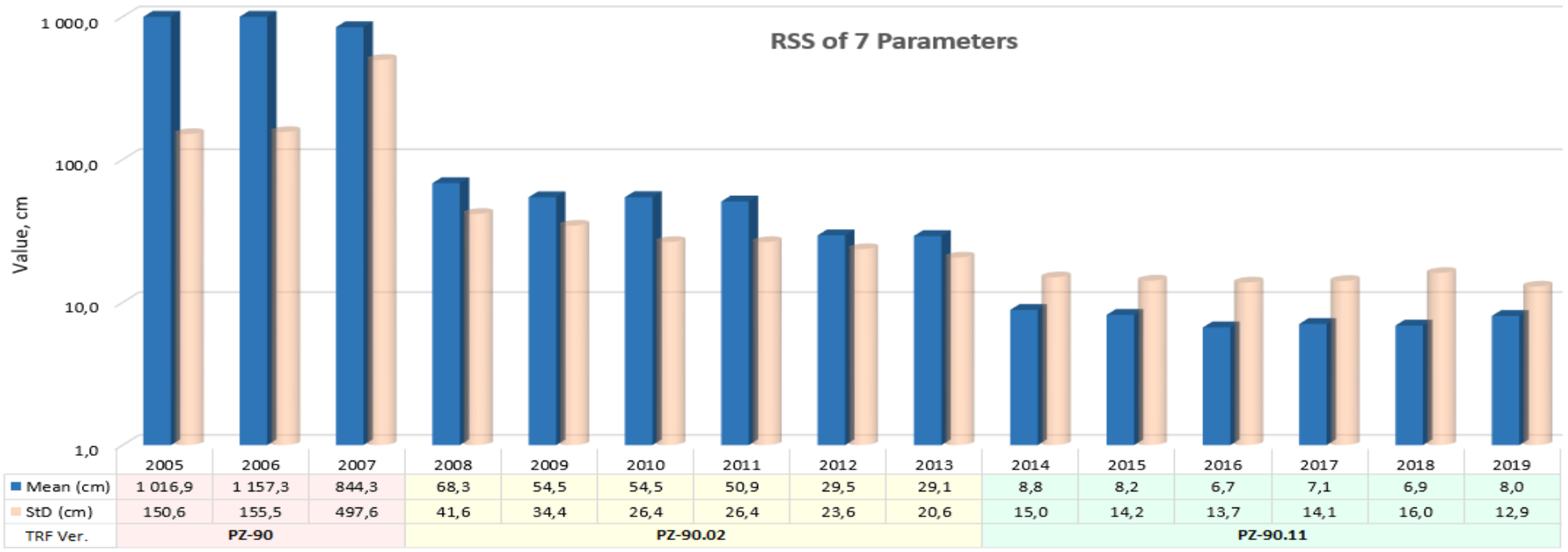
# Numerical Results of PZ-90 & ITRF Transformation Parameters through GLONASS Orbits: Mean Values and Standard Deviations (StD)

| TRF | Year | NUDS* | Value | ΔX (cm) | ΔY (cm) | ΔZ (cm) | $\omega_X$ (mas) | $\omega_Y$ (mas) | $\omega_Z$ (mas) | Scale (ppb) | $RSS_7$ (cm) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PZ-90 | 2005 | 172 | Mean | -0,2 | 0,9 | -68,3 | 30,08 | 11,42 | -326,89 | 2,16 | **1016,9** |
| | 2005 | 172 | StD | 9,6 | 10,1 | 70,6 | 15,90 | 25,03 | 30,89 | 2,14 | 150,6 |
| | 2006 | 365 | Mean | 0,0 | -0,4 | -75,4 | -6,59 | -4,82 | -373,81 | 1,67 | **1157,3** |
| | 2006 | 365 | StD | 8,8 | 9,5 | 73,6 | 21,64 | 27,40 | 26,98 | 2,83 | 155,5 |
| | 2007 | 365 | Mean | -0,1 | 1,3 | -39,8 | -0,60 | -0,23 | -273,03 | 1,05 | **844,3** |
| | 2007 | 365 | StD | 11,3 | 11,7 | 47,2 | 5,92 | 7,85 | 159,97 | 3,10 | 497,6 |
| PZ-90.02 | 2008 | 366 | Mean | 0,1 | -0,6 | 25,2 | 0,06 | 0,29 | -20,55 | 1,18 | **68,3** |
| | 2008 | 366 | StD | 5,6 | 5,9 | 18,8 | 3,39 | 3,29 | 10,70 | 3,07 | 41,6 |
| | 2009 | 365 | Mean | 0,2 | -0,9 | 19,7 | -0,20 | 0,10 | -16,40 | -5,67 | **54,5** |
| | 2009 | 365 | StD | 3,9 | 4,3 | 10,9 | 2,26 | 2,47 | 9,83 | 1,62 | 34,4 |
| | 2010 | 365 | Mean | 0,5 | -1,0 | 18,2 | 0,08 | -0,16 | -16,54 | -8,37 | **54,5** |
| | 2010 | 365 | StD | 2,3 | 2,3 | 10,0 | 1,72 | 1,87 | 7,42 | 1,43 | 26,4 |
| | 2011 | 365 | Mean | -0,3 | -0,4 | 18,6 | 0,26 | -0,29 | -15,22 | -8,53 | **50,9** |
| | 2011 | 365 | StD | 2,6 | 2,4 | 8,9 | 1,52 | 1,68 | 7,63 | 1,23 | 26,4 |
| | 2012 | 366 | Mean | -0,3 | -0,2 | 12,8 | 0,23 | -0,05 | -8,58 | 0,45 | **29,5** |
| | 2012 | 366 | StD | 2,6 | 3,5 | 6,8 | 1,27 | 1,41 | 6,77 | 6,69 | 23,6 |
| | 2013 | 365 | Mean | -0,4 | -0,1 | 11,4 | -0,16 | 0,06 | -8,63 | 4,49 | **29,1** |
| | 2013 | 365 | StD | 1,8 | 1,5 | 7,4 | 1,18 | 1,40 | 5,88 | 0,81 | 20,6 |
| PZ-90.11 | 2014 | 365 | Mean | -0,3 | -0,1 | 6,6 | 0,05 | 0,34 | 1,62 | 4,62 | **8,8** |
| | 2014 | 365 | StD | 1,5 | 1,5 | 5,3 | 0,78 | 0,91 | 4,31 | 0,80 | 15,0 |
| | 2015 | 364 | Mean | -0,1 | 0,0 | 7,6 | 0,18 | 0,22 | -0,28 | 4,30 | **8,2** |
| | 2015 | 364 | StD | 1,7 | 1,4 | 5,7 | 0,82 | 1,10 | 3,90 | 1,00 | 14,2 |
| | 2016 | 364 | Mean | 0,2 | 0,1 | 6,0 | 0,02 | 0,22 | 0,47 | 3,71 | **6,7** |
| | 2016 | 364 | StD | 1,6 | 1,5 | 5,1 | 0,90 | 1,04 | 3,84 | 0,85 | 13,7 |
| | 2017 | 363 | Mean | 0,1 | 0,0 | 6,7 | 0,06 | -0,12 | 0,16 | 3,55 | **7,1** |
| | 2017 | 363 | StD | 1,4 | 1,5 | 5,6 | 0,89 | 1,10 | 3,88 | 0,96 | 14,1 |
| | 2018 | 365 | Mean | 0,1 | 0,1 | 6,4 | 0,09 | -0,03 | -0,41 | 3,20 | **6,9** |
| | 2018 | 365 | StD | 1,6 | 1,5 | 5,9 | 0,96 | 0,97 | 4,57 | 0,88 | 16,0 |
| | 2019 | 295 | Mean | 0,2 | 0,0 | 7,8 | 0,15 | 0,04 | 0,30 | 2,58 | **8,0** |
| | 2019 | 295 | StD | 1,4 | 1,4 | 5,0 | 0,88 | 0,93 | 3,56 | 0,77 | 12,9 |

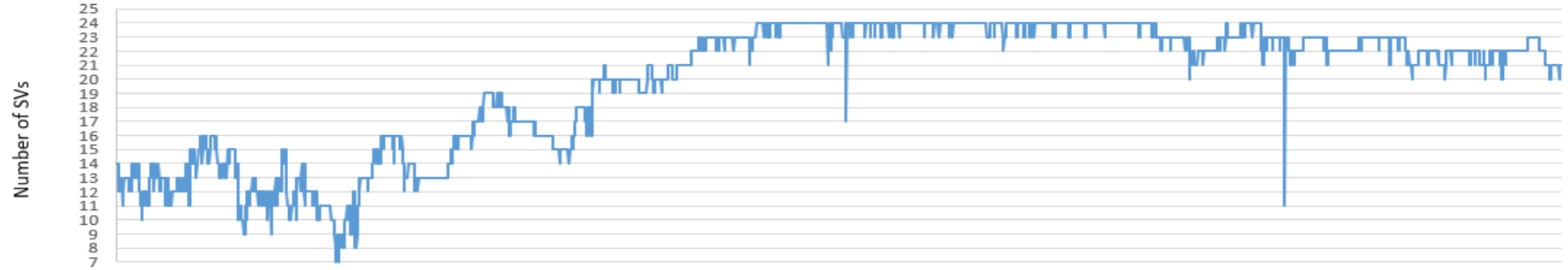
* Number of Used Dayly Solutions

# Evolution of PZ-90 TRF Realizations

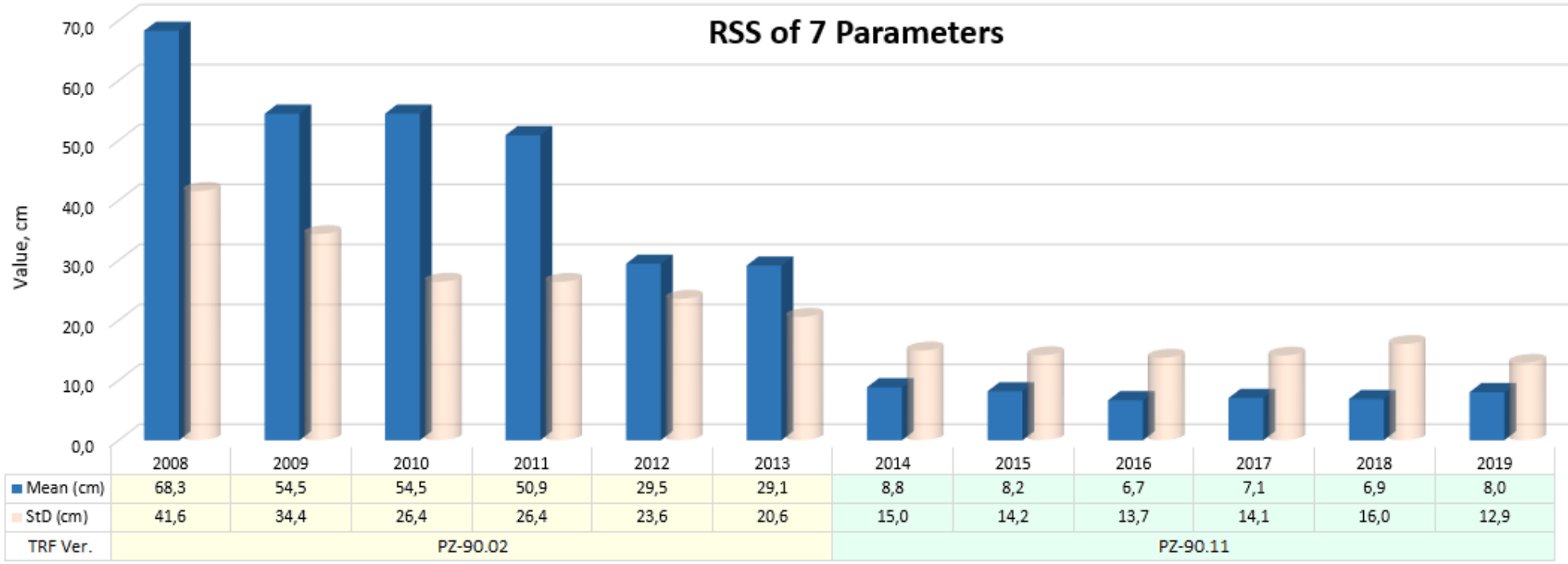
Logarithmic Scale



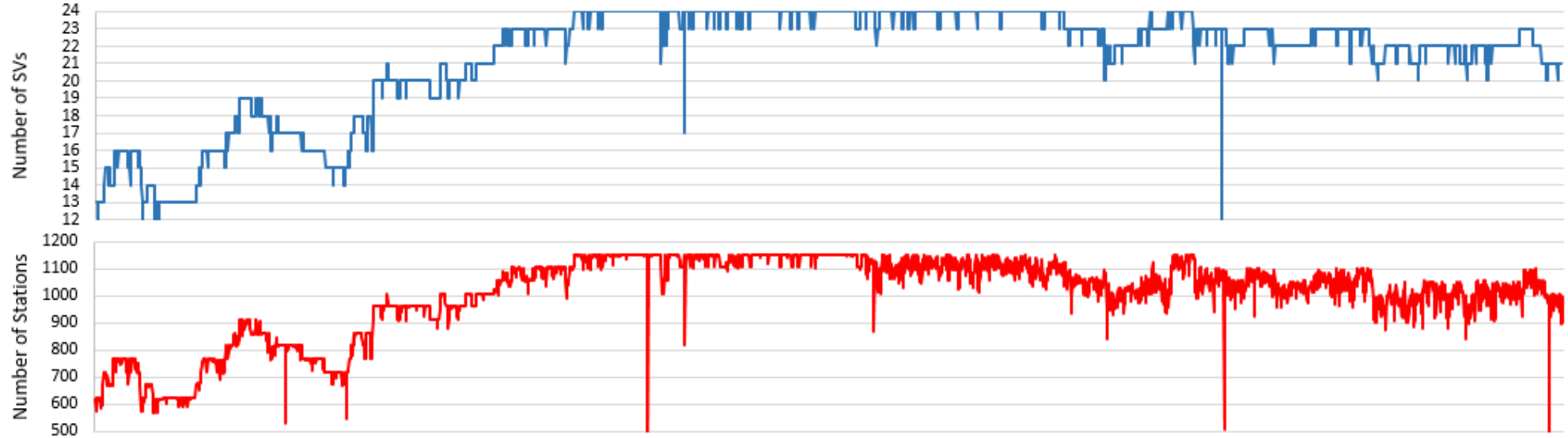

| | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean (cm) | 1 016,9 | 1 157,3 | 844,3 | 68,3 | 54,5 | 54,5 | 50,9 | 29,5 | 29,1 | 8,8 | 8,2 | 6,7 | 7,1 | 6,9 | 8,0 |
| StD (cm) | 150,6 | 155,5 | 497,6 | 41,6 | 34,4 | 26,4 | 26,4 | 23,6 | 20,6 | 15,0 | 14,2 | 13,7 | 14,1 | 16,0 | 12,9 |
| TRF Ver. | PZ-90 | | | PZ-90.02 | | | | | | PZ-90.11 | | | | | |




# Evolution of PZ-90 TRF Realizations (2)




| | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean (cm) | 68,3 | 54,5 | 54,5 | 50,9 | 29,5 | 29,1 | 8,8 | 8,2 | 6,7 | 7,1 | 6,9 | 8,0 |
| StD (cm) | 41,6 | 34,4 | 26,4 | 26,4 | 23,6 | 20,6 | 15,0 | 14,2 | 13,7 | 14,1 | 16,0 | 12,9 |
| TRF Ver. | PZ-90.02 | | | | | | PZ-90.11 | | | | | |



# Summary

- PZ-90.11 coincides with ITRF2014 at a level of less than 2 cm.
- PZ-90.11 accessible via broadcast GLONASS messages coincides with ITRF2014 at a level of less than 10 cm.
- GLONASS facilitates interoperability with other GNSS regarding to terrestrial reference frame.
- GLONASS provides access to terrestrial reference frame.