Session 2.2: Gravity and WHS

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Session 2.2 Program

Précis: *The gravity field is directly related to the structure of the Earth and how its mass is distributed. Every piece of mass creates a potential of gravity (geopotential) that drops off with distance. The cumulative effect of all these produces the Earth's gravity field. This session will focus on the relationship between various aspects of the Earth's gravity field such as the geoid, geopotentials, gravity, deflections of the vertical, and physical heights (e.g., above mean sea level). It covers different means of observing the gravity field and how they are combined to produce models for height determination both at global scales, such as the World Height System, and locally for National Vertical Datums.*

- **1030 – 1100 Session 2.2: Gravity and WHS**

- **1100 – 1130 Morning Tea and networking with Young Surveyors / Professionals**

  Restaurant, Cassa Geometri – *Sponsored by Cassa Geometri*

- **1130 – 1230** Continuation of **Session 2.2: Gravity and WHS**

Outline

- Introduction, outline, background: 5 slides
- Relationships between gravity, geopotential, DoV's & heights: 4 slides
- The gravity field spectrum -long wavelength, etc.: 4 slides
- Satellite gravity overview (GRACE/GOCE) - 3 slides
- Surface gravity (shipborne/land and relative/absolute): 2 slides
  - absolute meters: 2 slides
  - relative meters: 3 slides
  - Surface surveys (terrestrial & shipborne) – 10 slides
- The Effect of Terrain (RTM's/TC's): 3 slides
- Altimetric Anomalies: Mapping the Oceans from Space: 4 slides
- Aerogravity: Bridging the gap between Satellites and Ground: 6 slides

Outline

- Deflection of the Vertical: the impact on IMU's: 1 slide
- Point estimates of geoid height: GNSS on leveling: 2 slides
- Tying it all together: EGM's: 3 slides
- Focusing on the local picture: Regional Models: 3 slides
- World Height System vs. National Vertical Datums: 1 slide
- Using a Geoid model as a National Vertical Datum: 1 slide
- NAVD 88: the U.S. Vertical Datum: 3 slides
- Hybrid modeling using control data and a geoid: 4 slides
- Outlook for a future U.S. vertical datum: 1 slide
- Summary: 5 slides

Background

Dan Roman earned his Ph.D. at the Ohio State University and has been a Research Geodesist with the National Geodetic Survey since 1999. He is the team lead for Geoid Modeling and Research as well as the Principal Investigator for the Gravity for Redefinition of the American Vertical Datum (GRAV-D) Project. He developed GEOID99, GEOID03, GEOID06, GEOID09, and associated models.

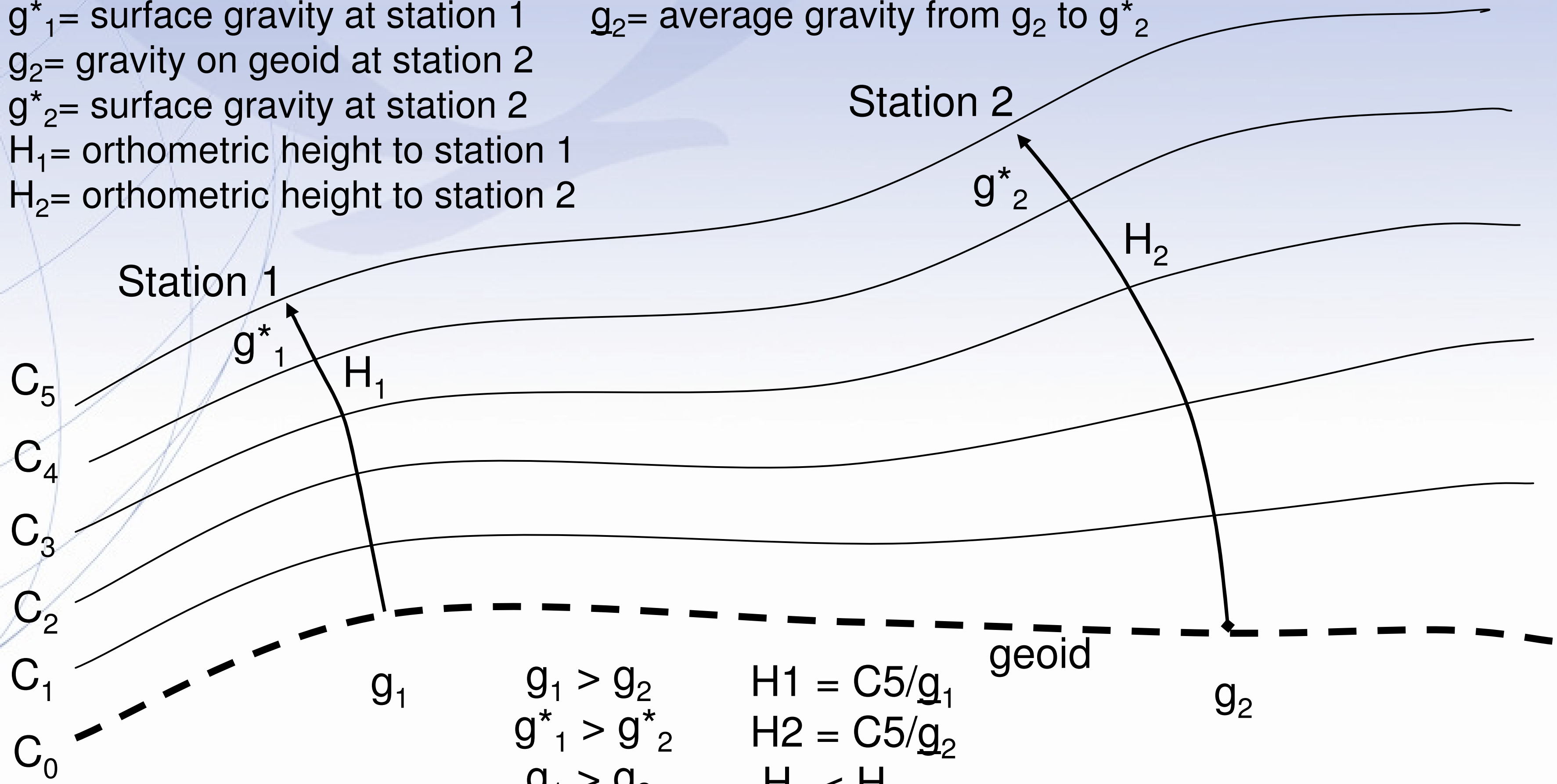
Gravity, Geopotential & Heights

- Mass attracts other mass
- M_E >>> any other mass
- Geopotential is 1/r
- Differential relationship
- Earth is more squashed
- So use *h* not *r*
- Then pick a datum (W_0)
- Physical heights relate to change in geopotential

- $g = Gm_1m_2/r^2$
- $g = GM\ (m)/r^2$
- $W = GM\ m/r$
- $g = \partial\ (W)/\ \partial r$
- geodetic not geocentric
- $g = \partial\ (W)/\ \partial h$
- $\int_0^P g\ \partial n = W_0 - W_P = C_P$
- $\Delta C_{b-a} = \sum_a^b h_i g_i$

g_1= gravity on geoid at station 1
g^*_1= surface gravity at station 1
g_2= gravity on geoid at station 2
g^*_2= surface gravity at station 2
H_1= orthometric height to station 1
H_2= orthometric height to station 2

$\underline{g}_1$= average gravity from g_1 to g^*_1
$\underline{g}_2$= average gravity from g_2 to g^*_2



$g_1 > g_2$ $\quad$ $H1 = C5/\underline{g}_1$
$g^*_1 > g^*_2$ $\quad$ $H2 = C5/\underline{g}_2$
$\underline{g}_1 > \underline{g}_2$ $\quad$ $H_1 < H_2$

Note that surface location of station 1 is closer to the geoid than station 2.
A steep gradient of geops indicates higher gravity – less steep indicates lower gravity.
The geops being farther apart beneath station 2 to reflect lower local mass and gravity.
Hence, H1 should be less than H2 – even though both have the same geopotential.

Gravity, Geopotential & Heights

- But the Earth is very big
- Need a good model: GRS80
- Work with residual values
  - Disturbing Potential
  - Gravity anomalies
  - Bruns Formula
  - Height relationships
  - Stokes' Formula

- Best to remove most signal
- It fits better than 99%
- Residual = actual – model

$T = W_P - V_P$

$\Delta g = g_P - \gamma_Q = \partial (T) / \partial h - 2T/h$

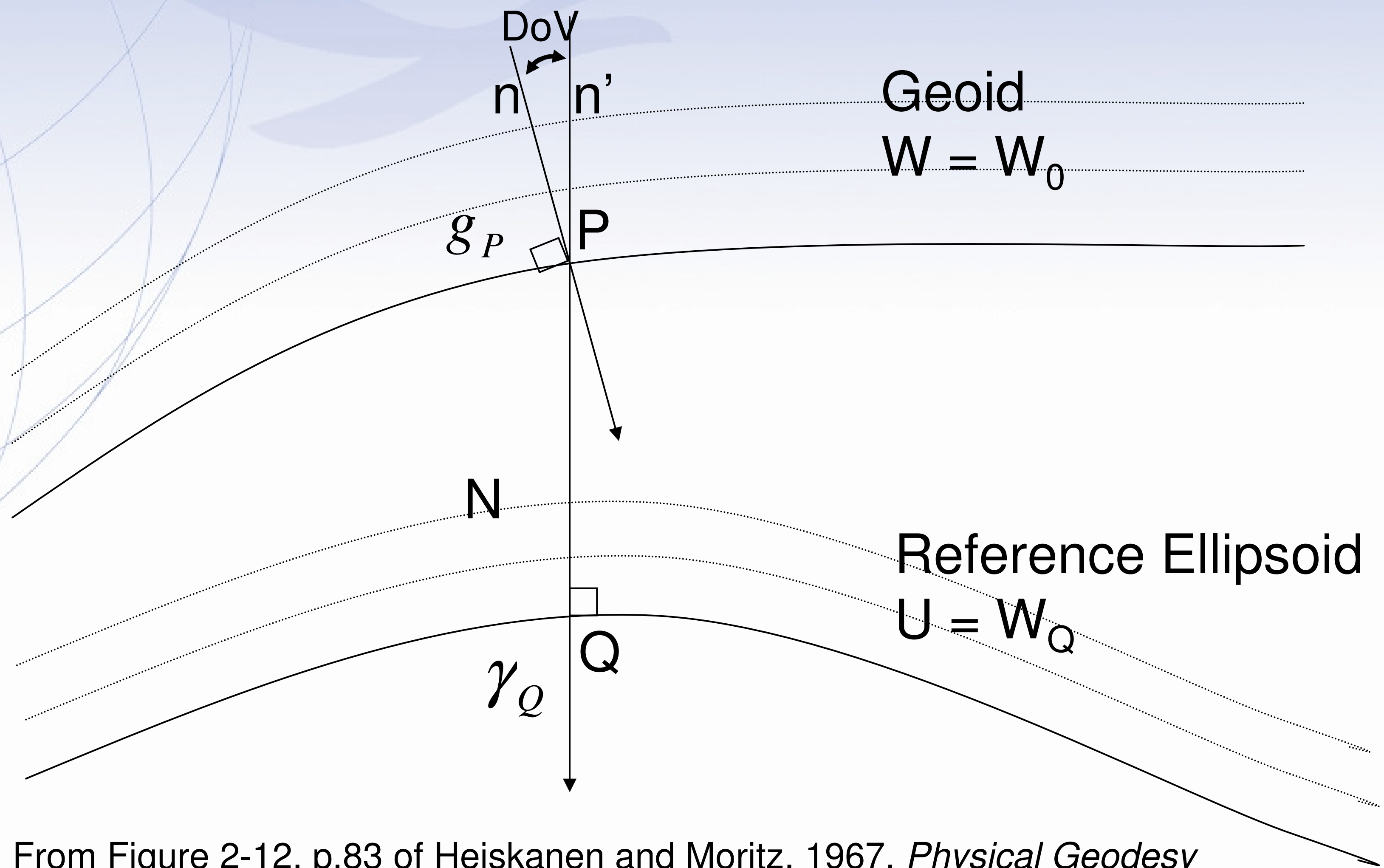
$N = T / g$

$h = H + N$

$$N = \frac{R}{4\pi G} \iint_{\sigma} \Delta g S(\psi) d\sigma$$

$$S(\psi) = \frac{1}{\sin\left(\psi/2\right)} - 6\sin\left(\frac{\psi}{2}\right) + 1 - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)$$

Gravity, Geopotential & Heights

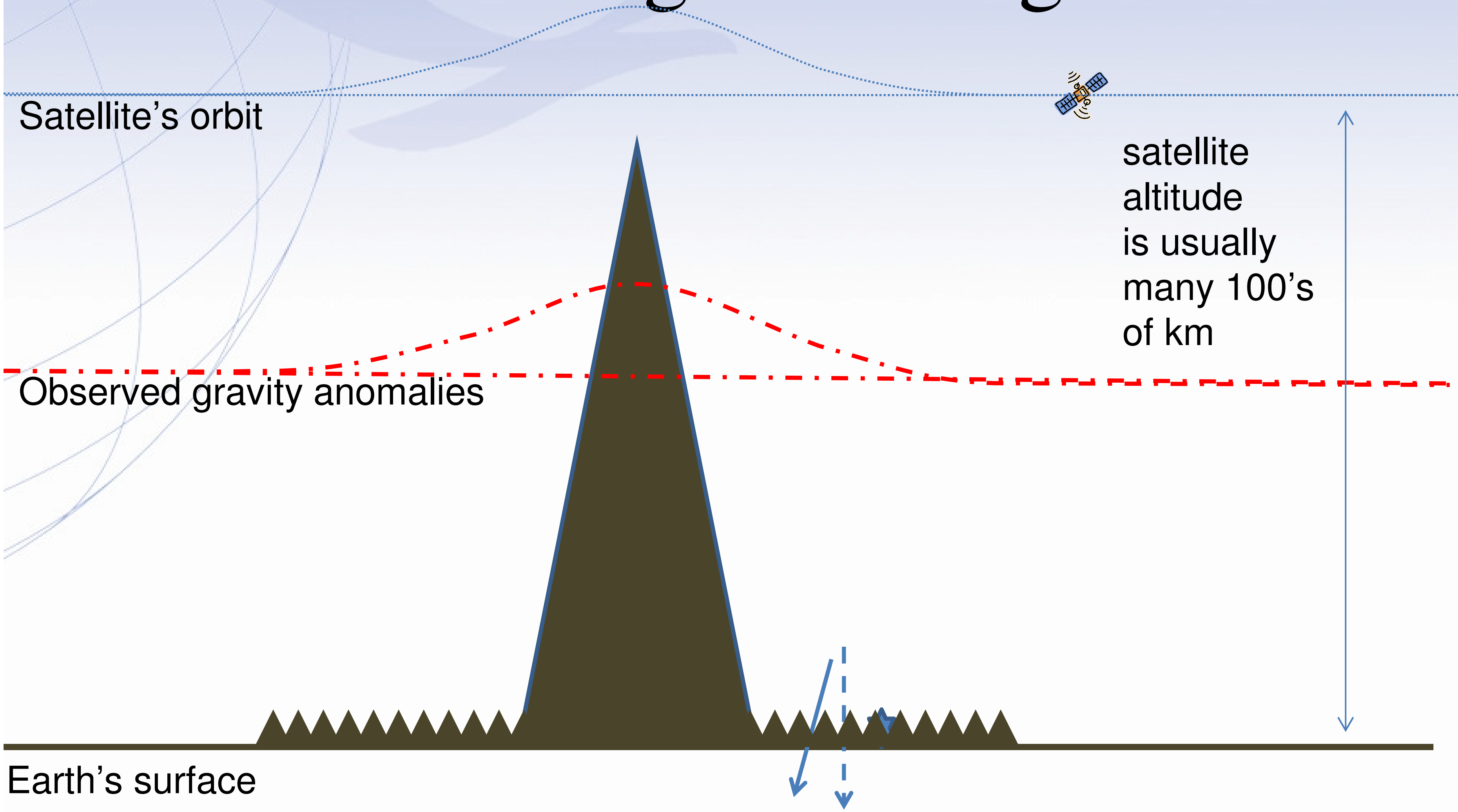


From Figure 2-12, p.83 of Heiskanen and Moritz, 1967, *Physical Geodesy*

The Gravity Field Spectrum

- Concepts of scale
  - Long wavelengths: satellites
  - Short wavelengths: terrain
- Degree-Spectrum plot
  - Satellite gravity models
  - Aerogravity
  - Surface gravity observations
  - Terrain modeled gravity

What does long wavelength mean?



What does short wavelength mean?

Dealing with residuals

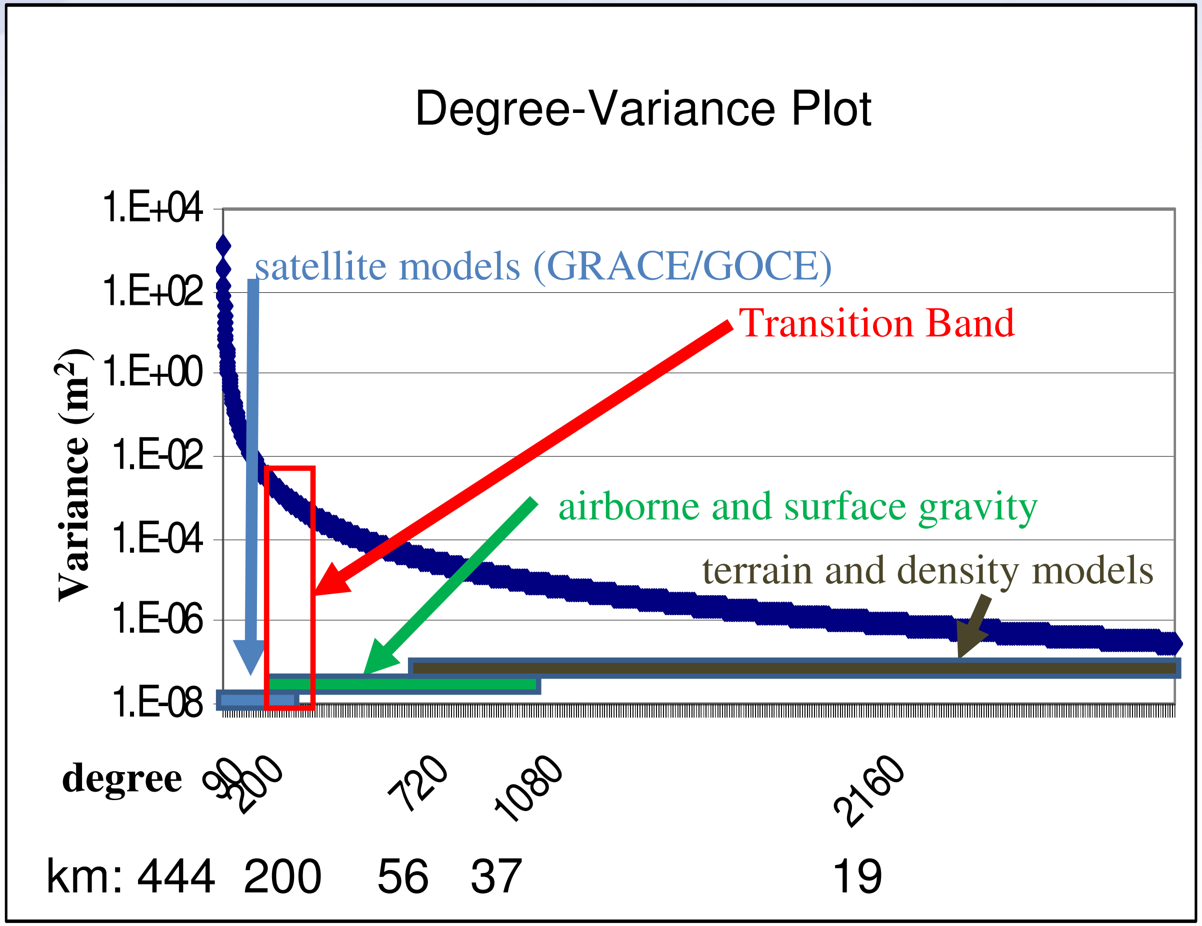
- gravity anomalies
- $\Delta g = g_P - \gamma_Q$
- Most signal removed (γ)
- Accounts for degree 2
- Leaves everything else
- Satellite models > 200 km
- Terrestrial gravity coverage not likely sufficient
- Must use DTM/DEM's

Dealing with mountains

- Residual Terrain Model
  - Models the terrain in SHM/EHM
  - Used in EGM2008 to 5'
  - Use SRTM 3" to get 3"-5'
  - 90 meter resolution
  - Signal < 3" affects Δg's
- Terrain Corrections
  - Used more with Stokes approach
  - Removes impact on gravity obs.
  - Then you make the geoid model

The Gravity Field Spectrum

- SHM/EHM
- Inverse to scale
- d. 2 => Ellipsoid
  - 2 oscil. in circle
- Earth is 40,000 km round
- d. 360 is 1/360th of that or 111 km
- d. 2160 is 18 km



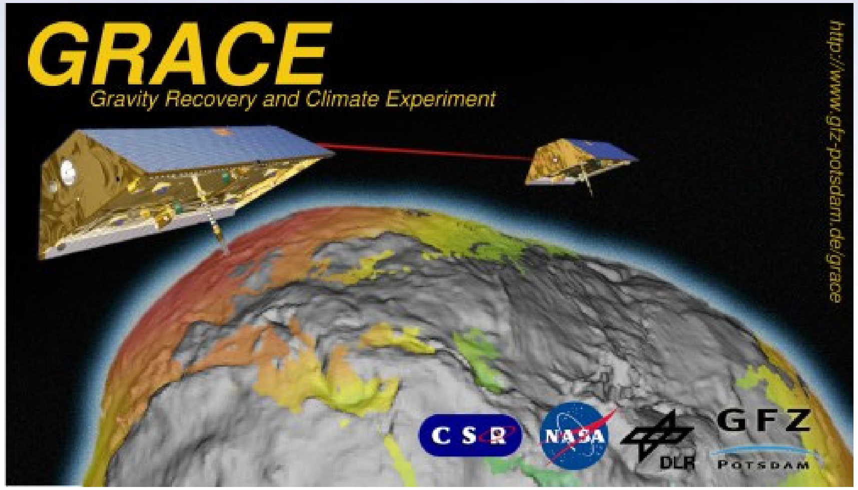
Satellite Geodesy

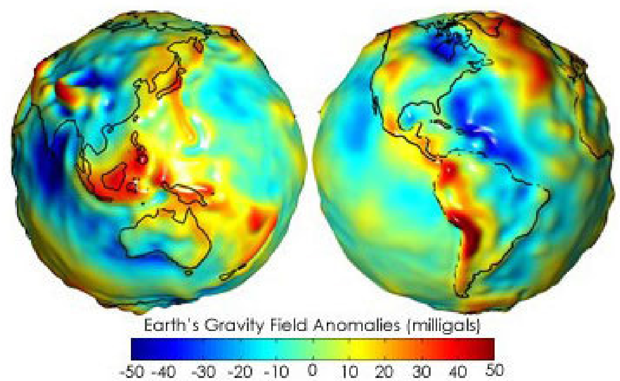
- Started with tracking satellites in orbit (60's)
- First dedicated gravity mission was CHAMP
- GRACE was next (10 years on and still going)
- GOCE flying (20 months)
- Likely GRACE II/GFO & follow-on ESA missions

- Best approaches involve
  - GNSS receivers: hi-low
  - Low-low tracking (GRACE)
- Basic idea: orbital changes arise from gravity changes
- NOTE: gravity is observed in orbit <u>not</u> on the ground
- Also, orbital height is a function of sensitivity to scale (attenuation)

Gravity Recovery and Climate Experiment (GRACE)

- PI's are Tapley (CSR U. of Texas) and Reigber (GFZ)
- Dual Satellites
  - Orbital tracking (350 km)
  - Satellite-to-Satellite (lo-lo, hi-lo)
- Polar orbit – solves for deg. 2
  - Previous solution gap > ±83
  - Solutions to deg. 120
  - Generally accepted to deg. 60-90
- Serves as base level for global gravity & geoid height models

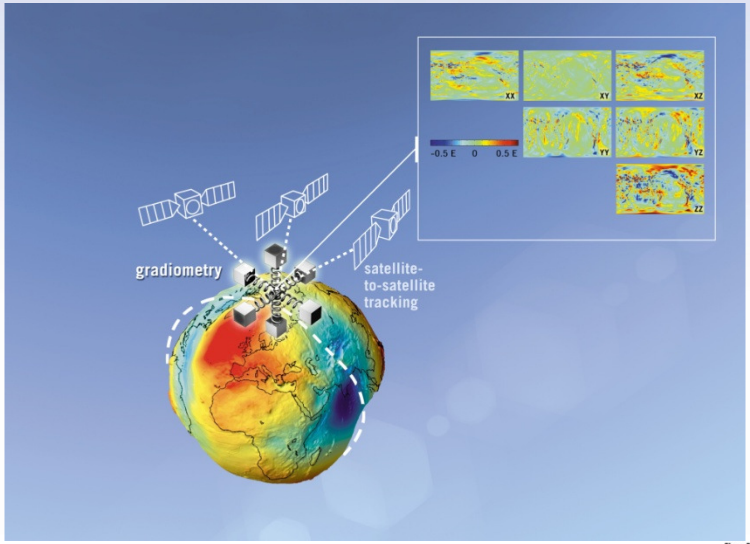


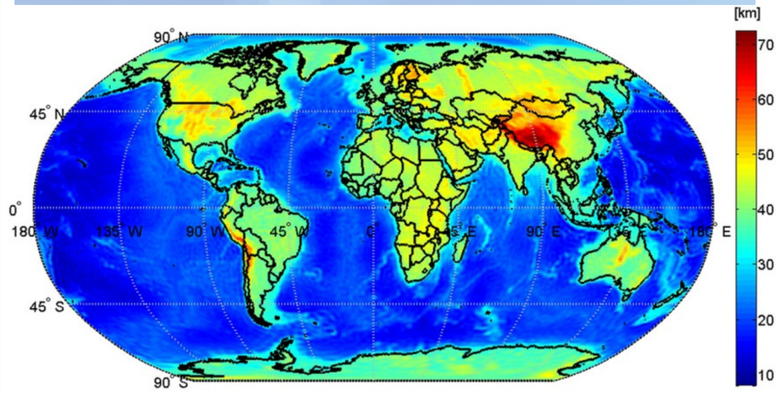


http://www.csr.utexas.edu/grace/

Gravity field and steady-state Ocean Circulation Explorer (GOCE)

- Mission Scientists: Drinkwater & Haagmans
- 3x Accelerometer pairs
  - Determines gradient ($\Delta(\Delta W)$)
  - Used to integrate Δg to surface
- Orbital tracking (260 km)
  - Satellite is compact & heavy
  - Short life expectancy (20 m.)
  - Resolves gravity to d.150 now
  - Ultimately will go to d. 200+





http://www.esa.int/SPECIALS/GOCE/index.html

From Satellite to Surface Gravity Observations

- Gravity meter types
- Surface gravity observations
- Examples from U.S data sets
  - Coverage/gaps
  - Systematic errors
  - Intra-survey errors
  - Inter-survey errors

Surface Gravity

- Absolute Meters (US$500k or more)
  - Terrestrial only – needs a lot of stability
  - National and international gravity networks
- Relative Meters (US$100k-US$1,000k)
  - Pendulums, springs with proof mass, SG
  - Uses: terrestrial, shipborne, airborne, & satellite
  - Most common type available and source of data
  - Yields difference in gravity between points

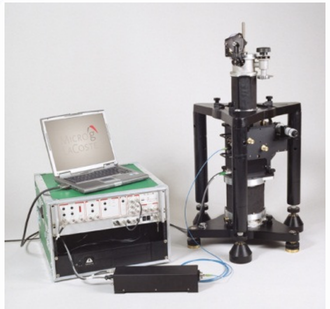
Absolute Gravity Meters

- Drop of proof mass (some are rise-fall tests)
- Interferometry of fringes to check acceleration
- Used to establish control for relative surveys
- Useful for work requiring extreme accuracy
- Manufacturers include
  - Micro-g LaCoste (Boulder, CO, USA)
  - Some academic & research models in development

Absolute Gravity Meters from Micro-g LaCoste

http://www.microglacoste.com/absolutemeters.php info@microglacoste.com

Meter	Accuracy	Precision	Repeatability	Temp Range	DC	AC	Outdoor Operation
	μGal	μGal/√(Hz)	μGal	°C			
FG-L	10	100	10	15 to 30		×	
A-10	10	100	10	-18 to +38	×	×	×
FG-5(X)	2	15	1	15 to 30		×	



Relative Gravity Meters

- Original models include pendulum
  - Change in gravity proportional to change in period
- Other models include mass on spring, zero-length spring, & magnetically-suspended mass
- Manufacturers include
  - Micro-g LaCoste (Boulder, CO, USA)
  - Bell Geospace (Houston, TX, USA)
  - ZLS Corporation (Austin, TX, USA)

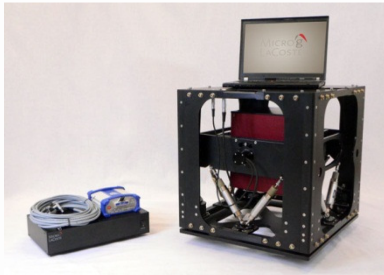
Relative Gravity Meters

- L & R Meter Service, LLC (Lexington, TX, USA)
- GWR Instruments, Inc. (San Diego, CA, USA) {SG}
- Neese Exploration (Haliburton) (Richmond, TX, USA)
- Fugro Gravity & Magnetics Services (worldwide)
- Sander Geophysics (Ottawa, Ontario, Canada)
- Canadian Micro Gravity (Toronto, Ontario, Canada)

Relative Gravity Meters from Micro-g LaCoste

- Land Meters
  - LaCoste-Romberg
  - Scintrex CG-5
- Air Meters
  - Air/Sea Gravity System II
  - Turnkey Airborne Gravity System
  - System 6 Dynamic Gravity Meter





http://www.microglacoste.com/relativemeters.php

Surface Gravity Surveys

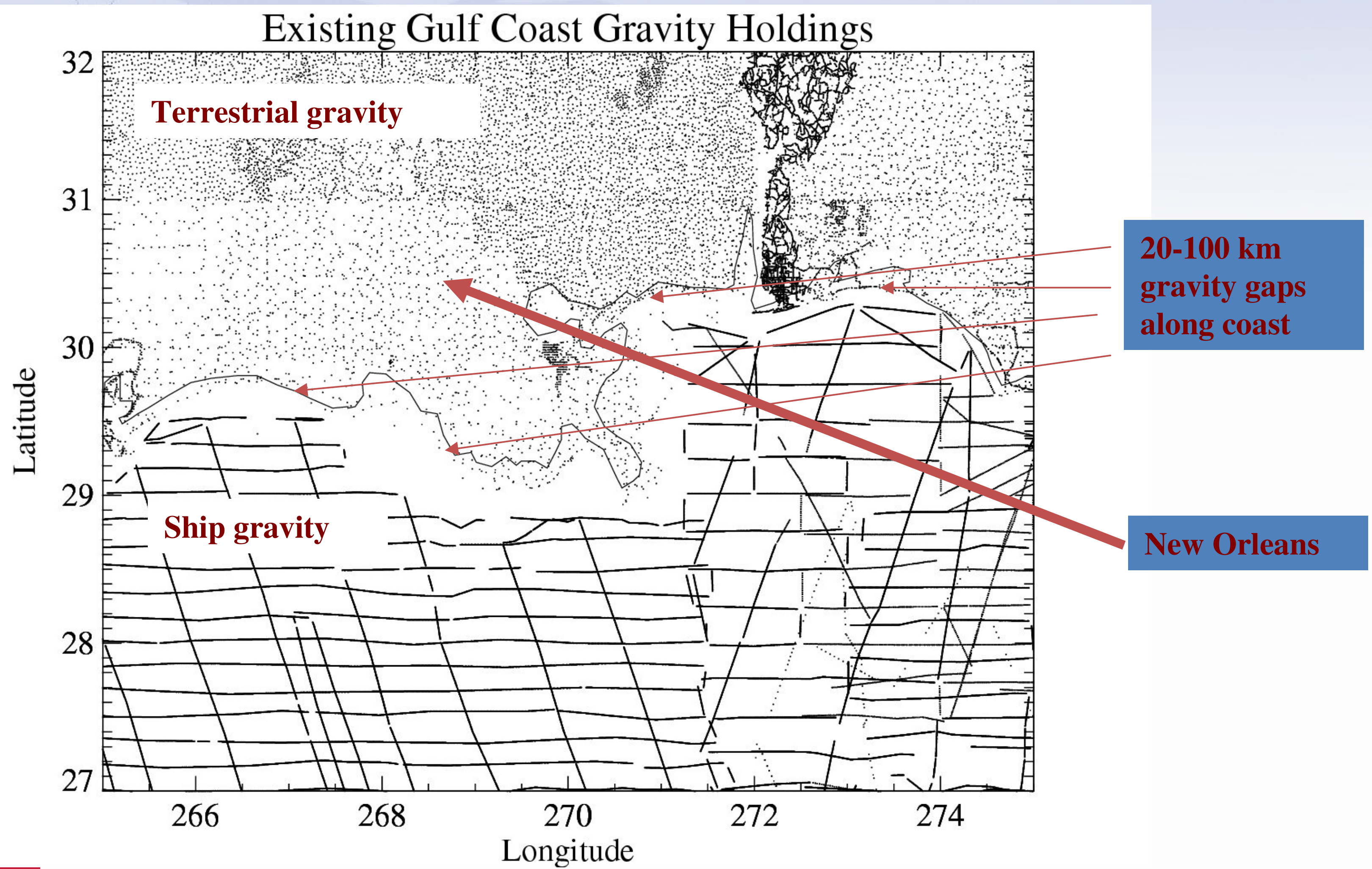
Terrestrial

- Many different sources
- Uncertain corrections
- Potential for biases is big
- Individual surveys are generally smaller than the "state" level
- Better distribution generally
- Equipment
  - Portable g-meters
  - GPS or scaled from map

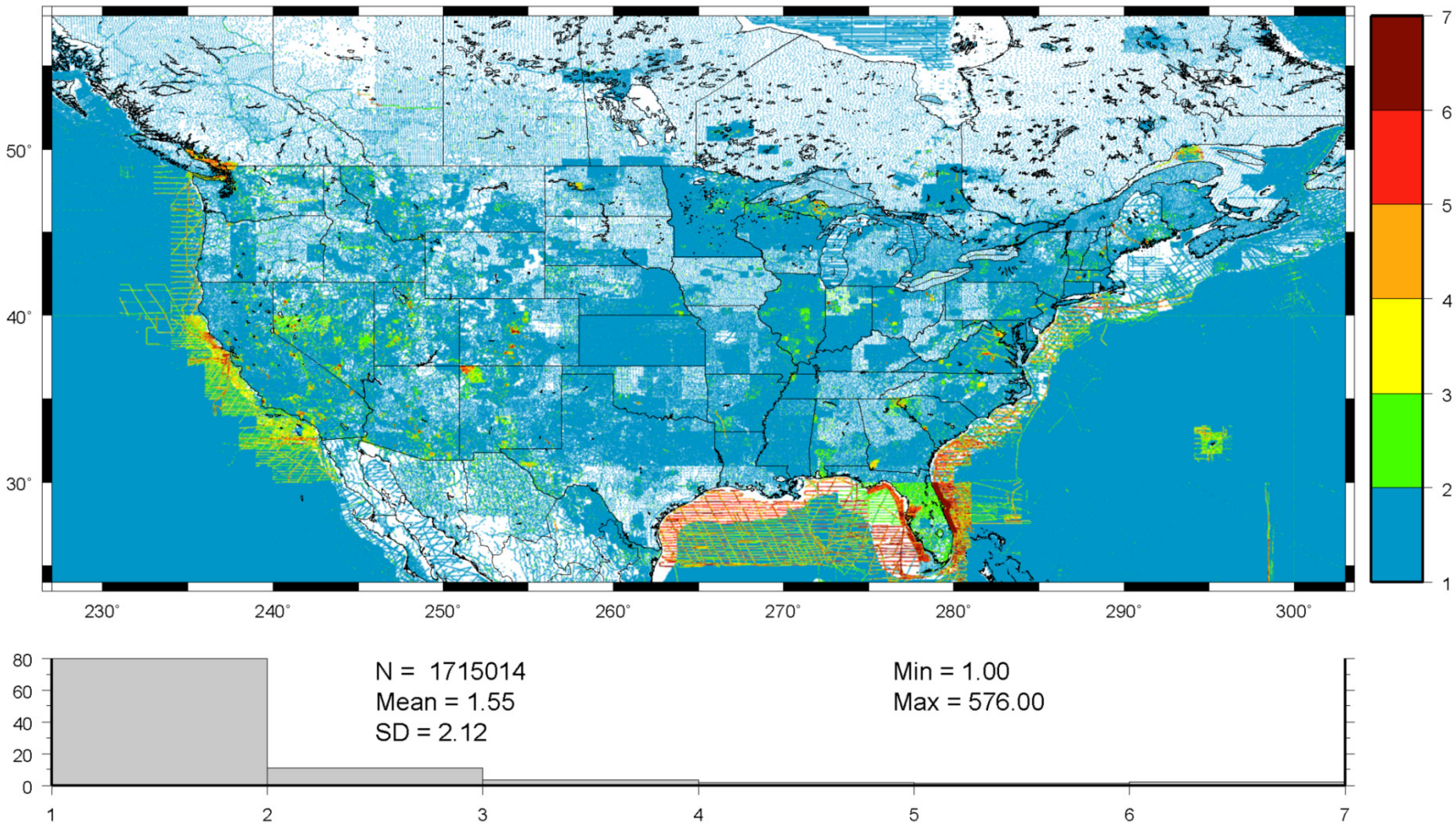
Shipborne

- Usually fewer sources
- More corrections required
- Internal consistency may be a problem (drift)
- Blown base ties can create huge systematic effect
- Greater sampling along track
- Equipment
  - Bell meter or Air/Sea II
  - GPS & IMU

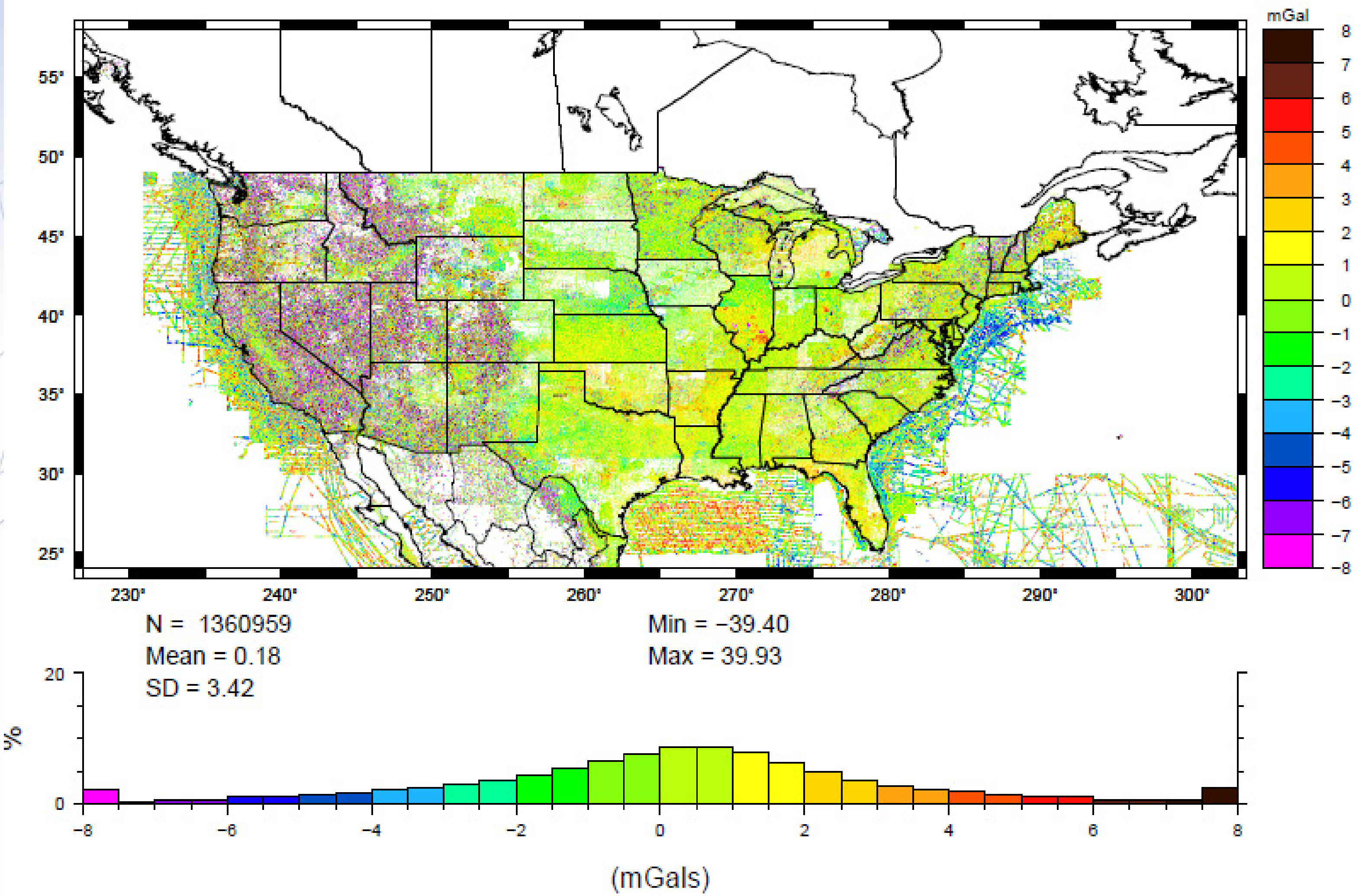
Near Shore Data Gaps



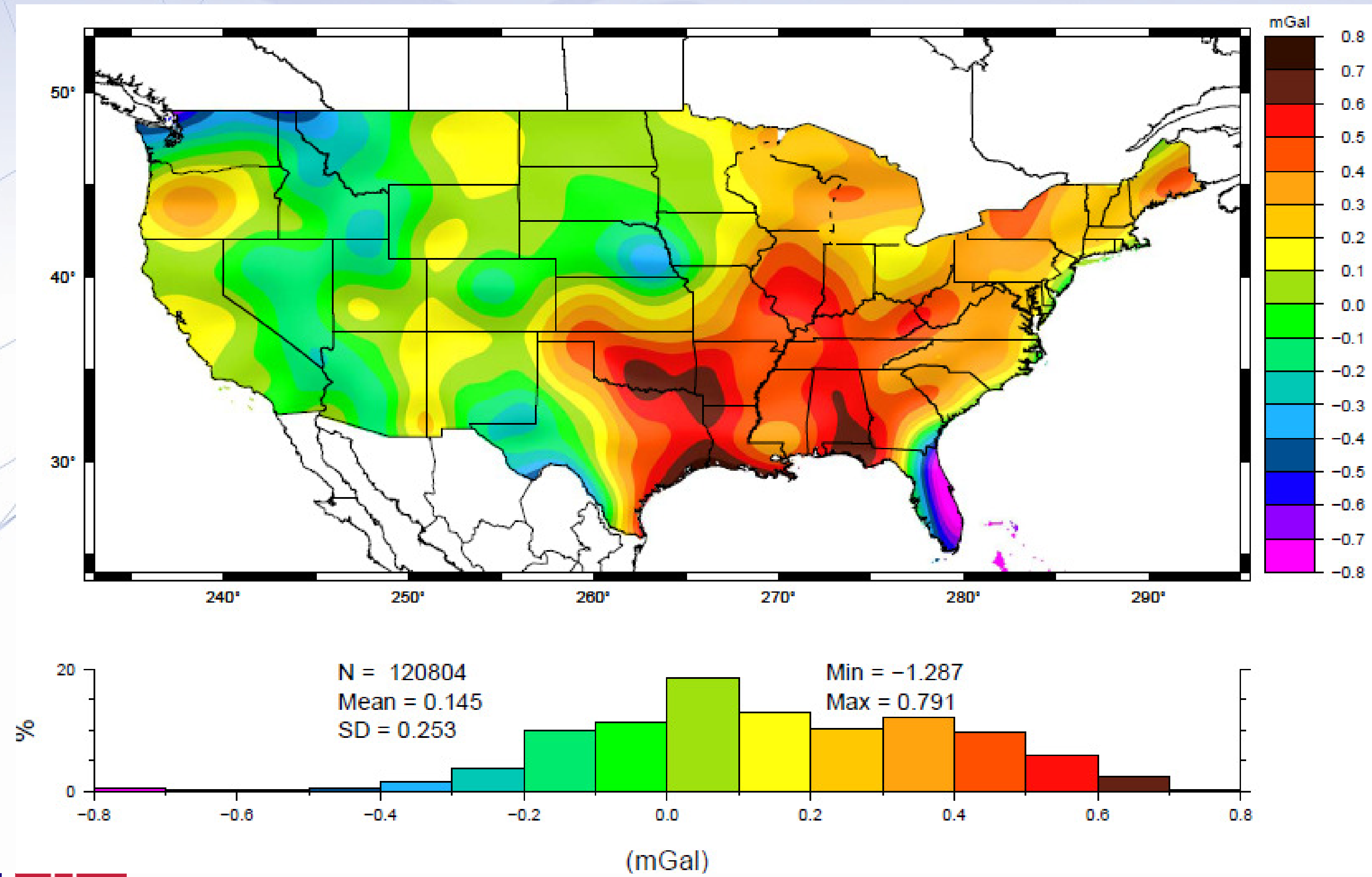
Number of Gravity Points Per 1x1 Minute cell



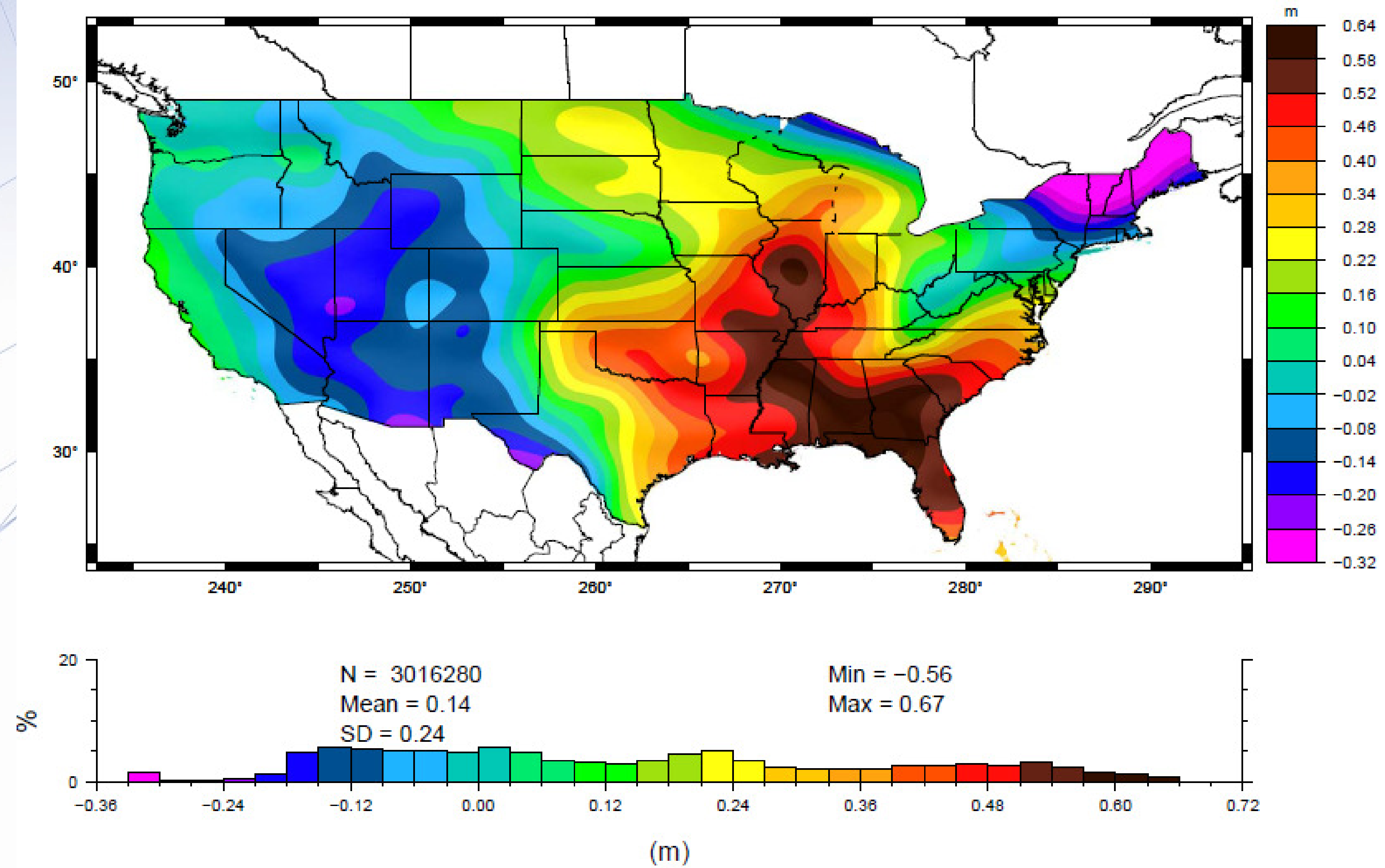
Residual gravity data for CONUS



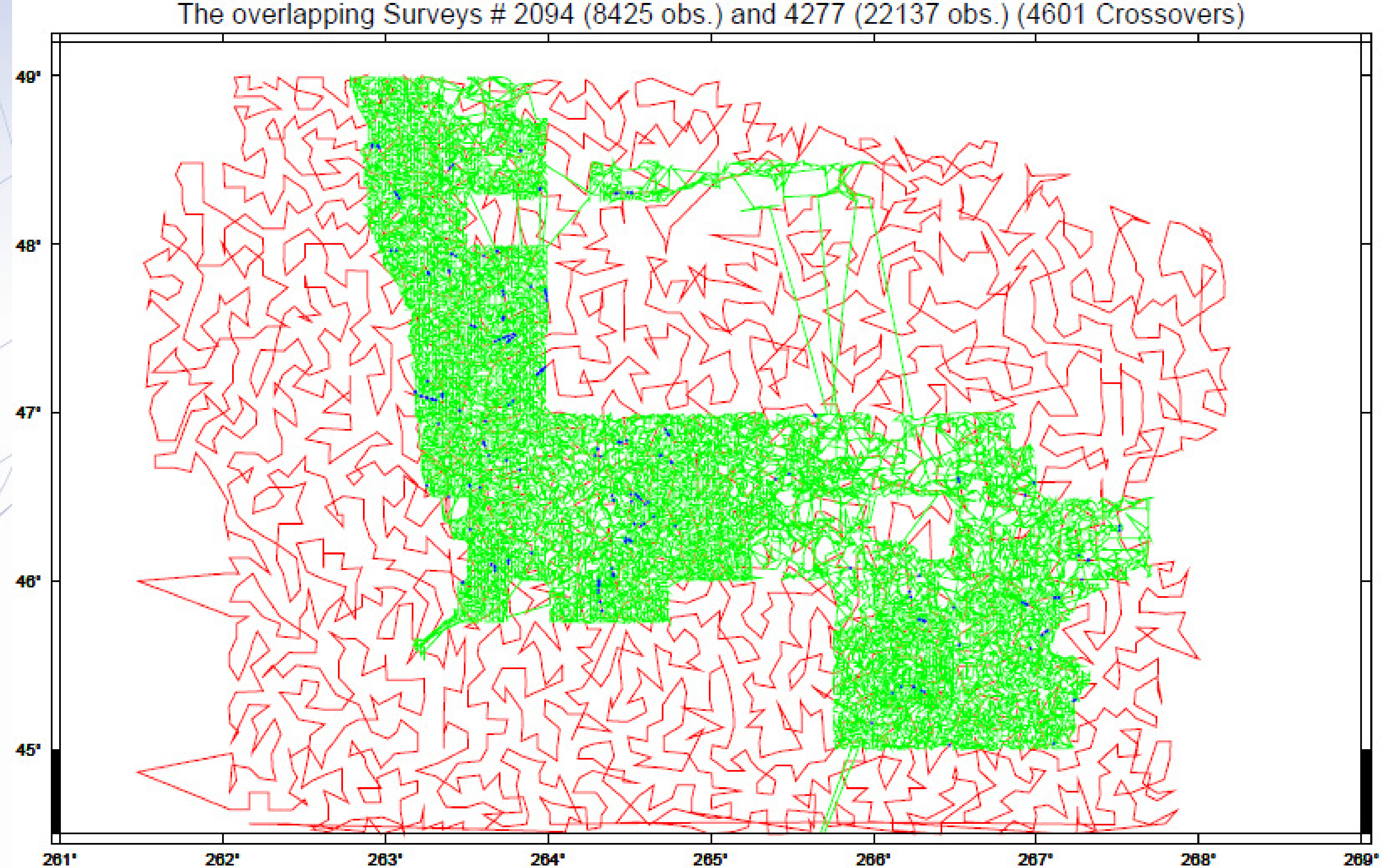
Long wavelength (deg 2-120) gravity errors



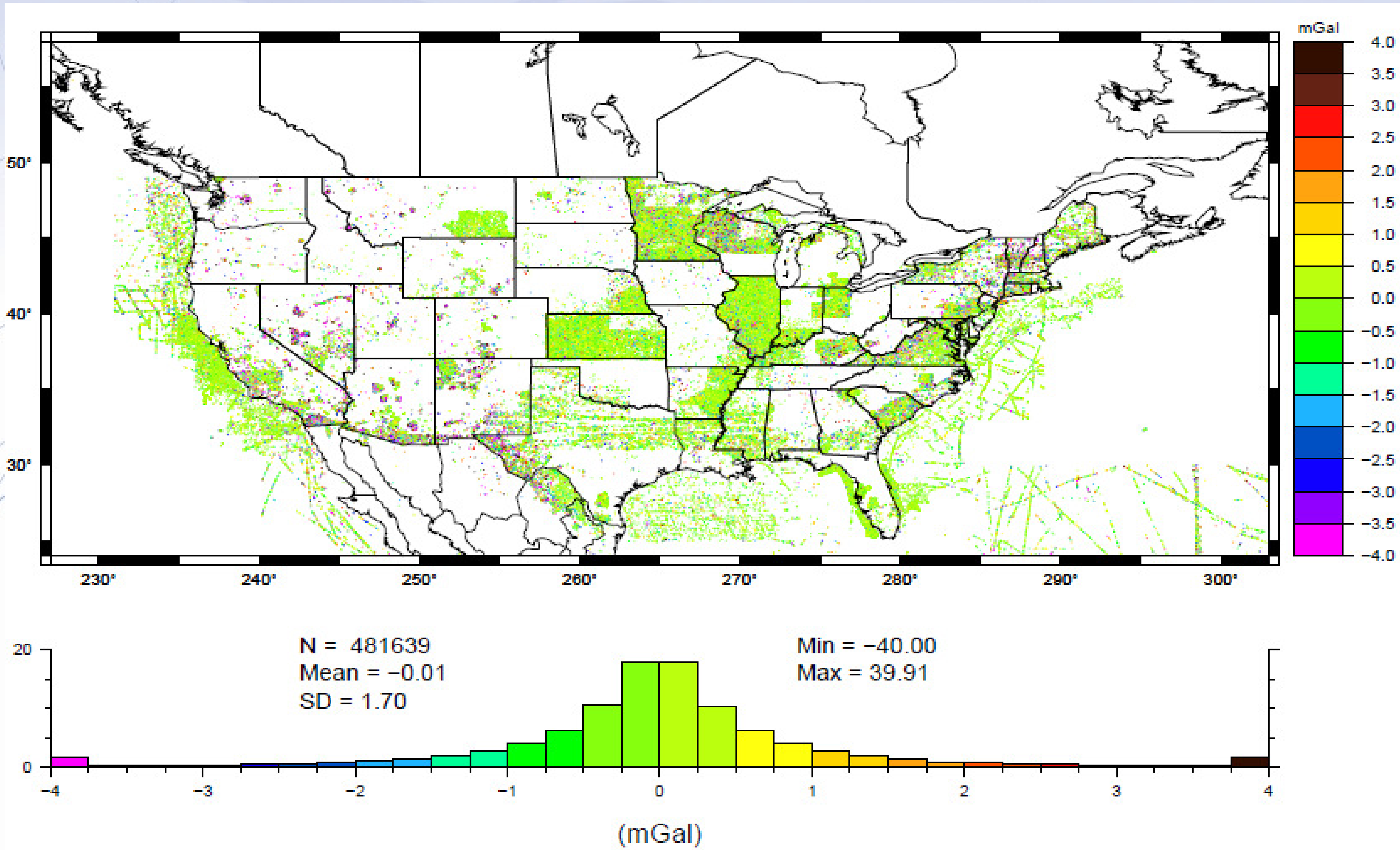
Long wavelength (deg 2-120) geoid errors



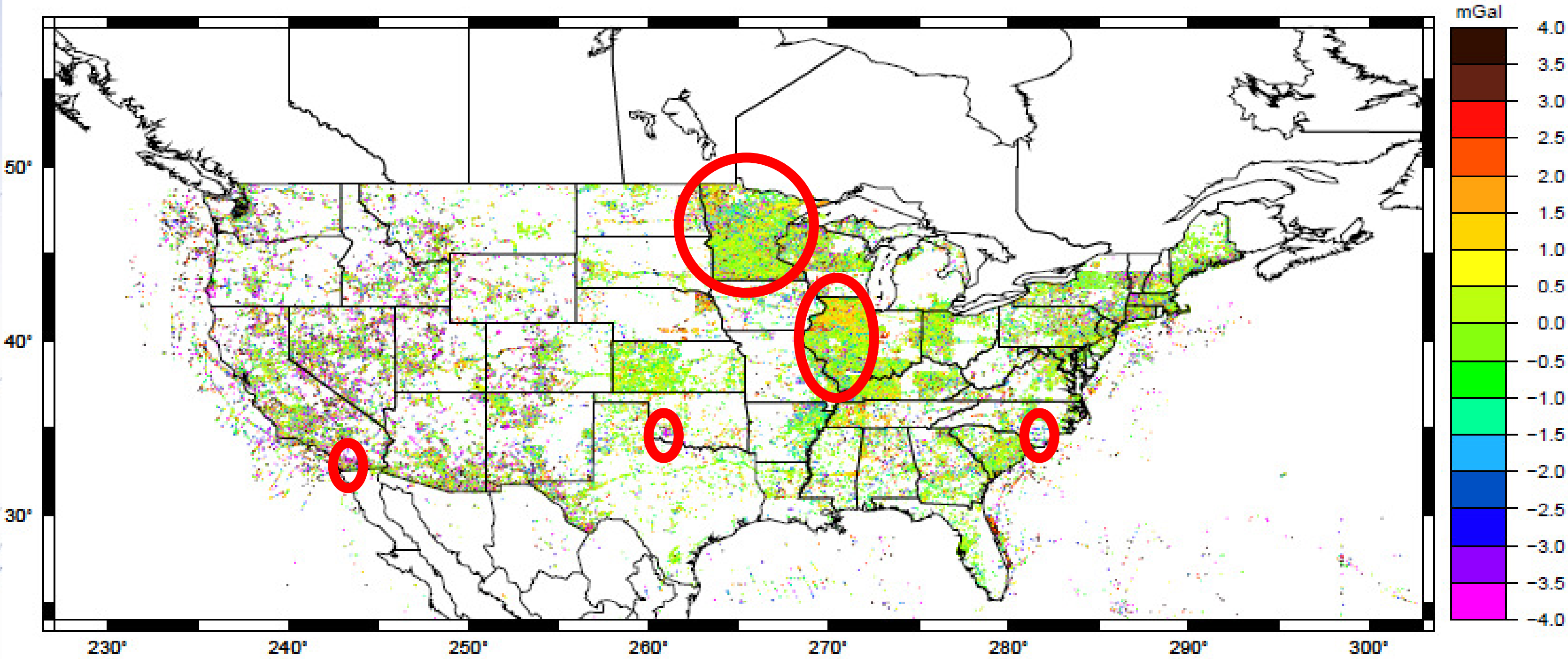
Surveys 2094 (red) and 4277 (green)

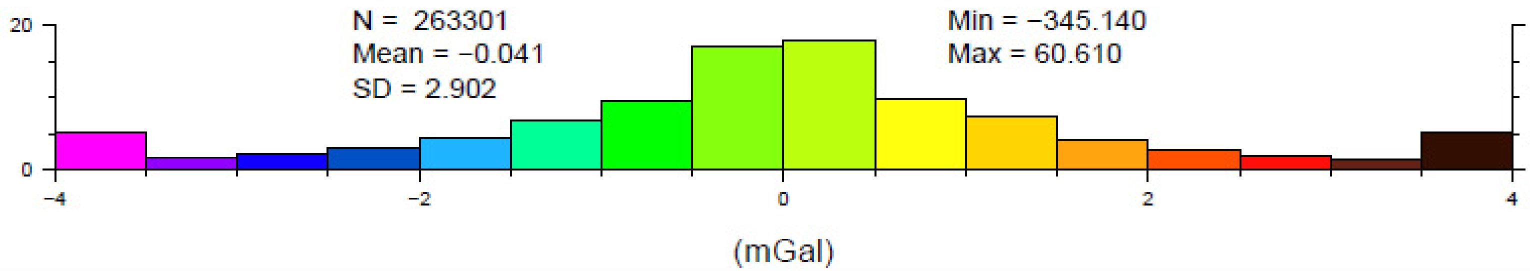


Internal Crossovers Between 1489 Surveys

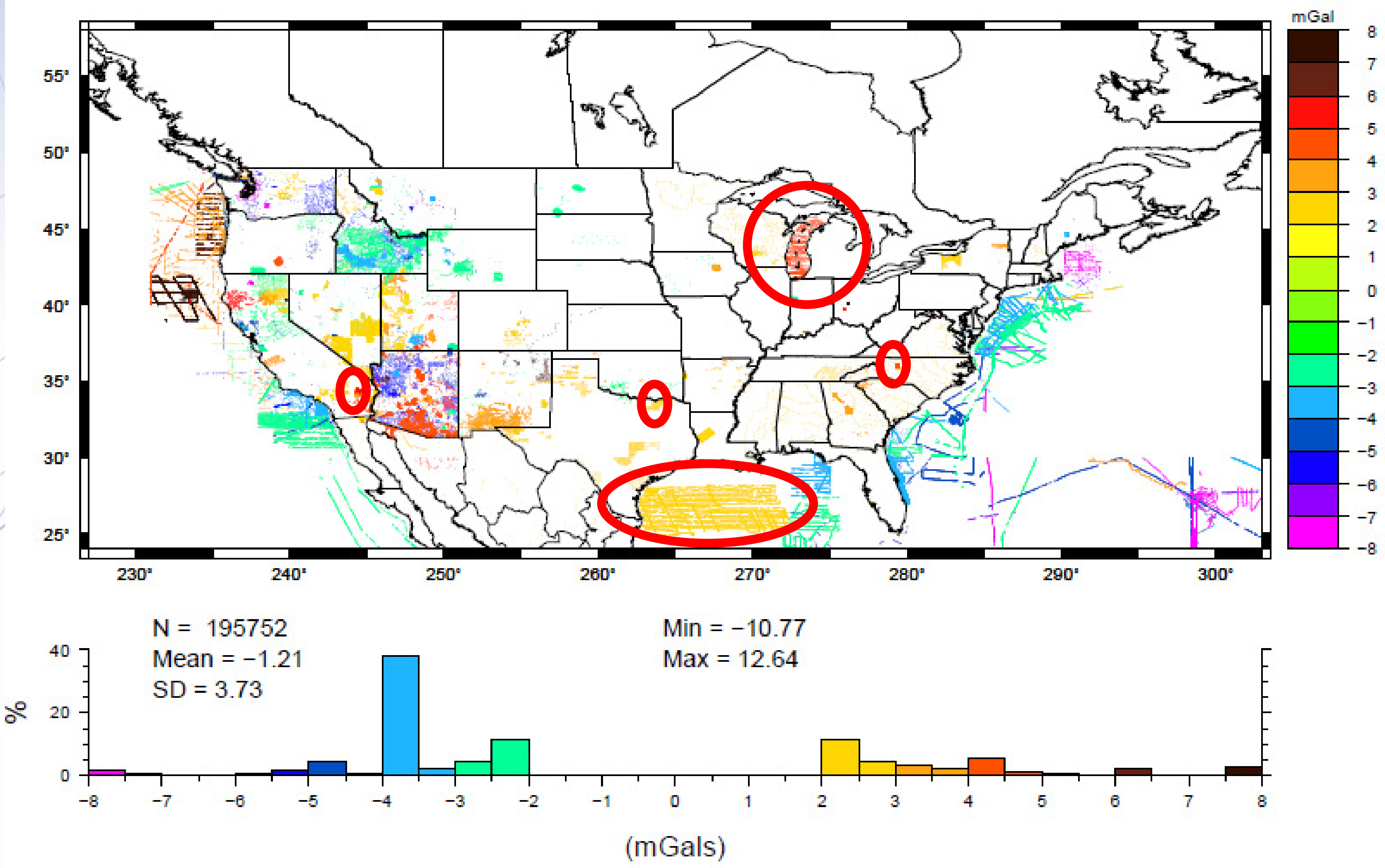


External Crossovers Between 1489 Surveys





244 Surveys with Significant Biases



The Effects of Terrain on the Short Wavelength (λ) Gravity Field

DEM/DTM/DTED

- Bare Earth model desirable
- Consistency is important too
- Sample interval varies
- Lidar is best but coverage is incomplete at national scale
- NED/CDED differences
- Satellites give near global coverage but may have gaps
- SRTM 3", ASTER, etc.

DDM

- Less significant than DEM's
- $\Delta\rho_{rock/air} >> \Delta\rho_{rock/rock}$
- Biggest impact: short λ
- Satellites capture long λ
- Most geodetic models use uniform density (ρ)
- DDM would provide better lateral/vertical variations

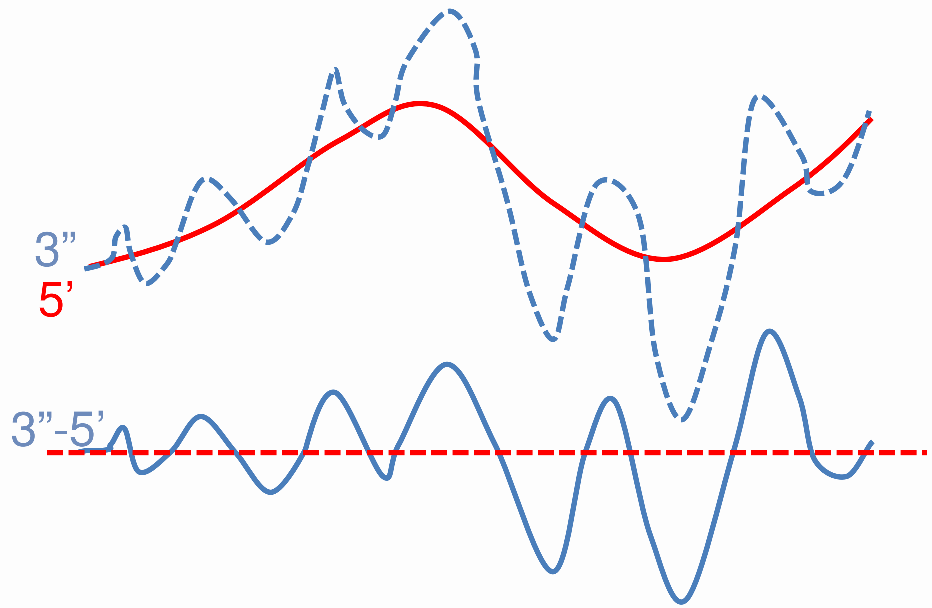
Residual Terrain Model (RTM)

General Concept

- Models the gravity effect of the variations in terrain
- Applied to gravity field model to remove effect
- EGM2008 has 5'- global
- Must account for more
- Not using 3"-5' RTM made 500,000 points too "noisy"
- Must develop residual RTM

3"-5' RTM Effects

- Interpolate 5' to 3" points
- Subtract for residual RTM



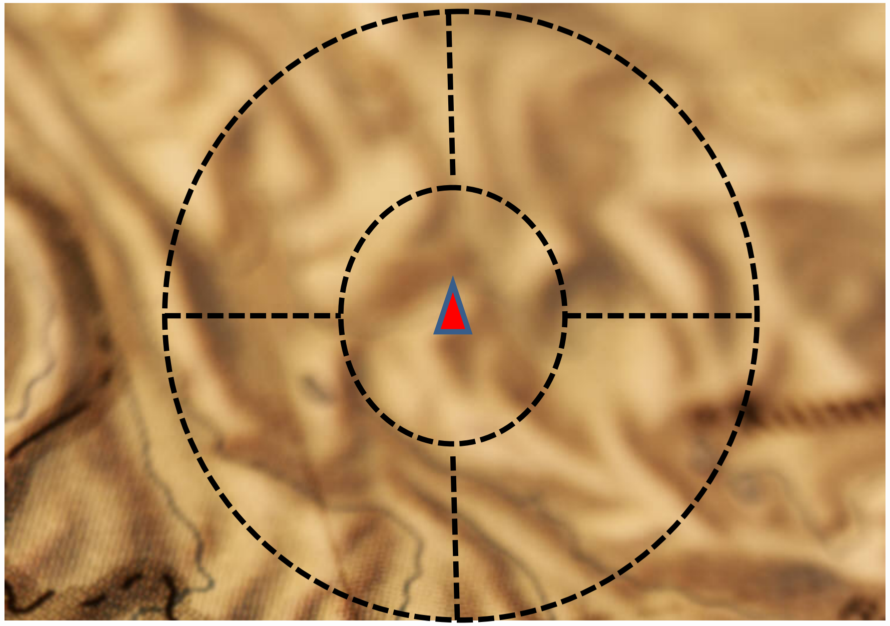
Terrain Corrections

General Concept

- Model the impact of terrain on gravity observations
- Correction is always positive
- A mountain will pull gravity vector towards it
- A canyon will deflect gravity vector away from it
- Net effects of both is to reduce observed gravity

Application

- Smoothes the gravity field for Helmert Condensation

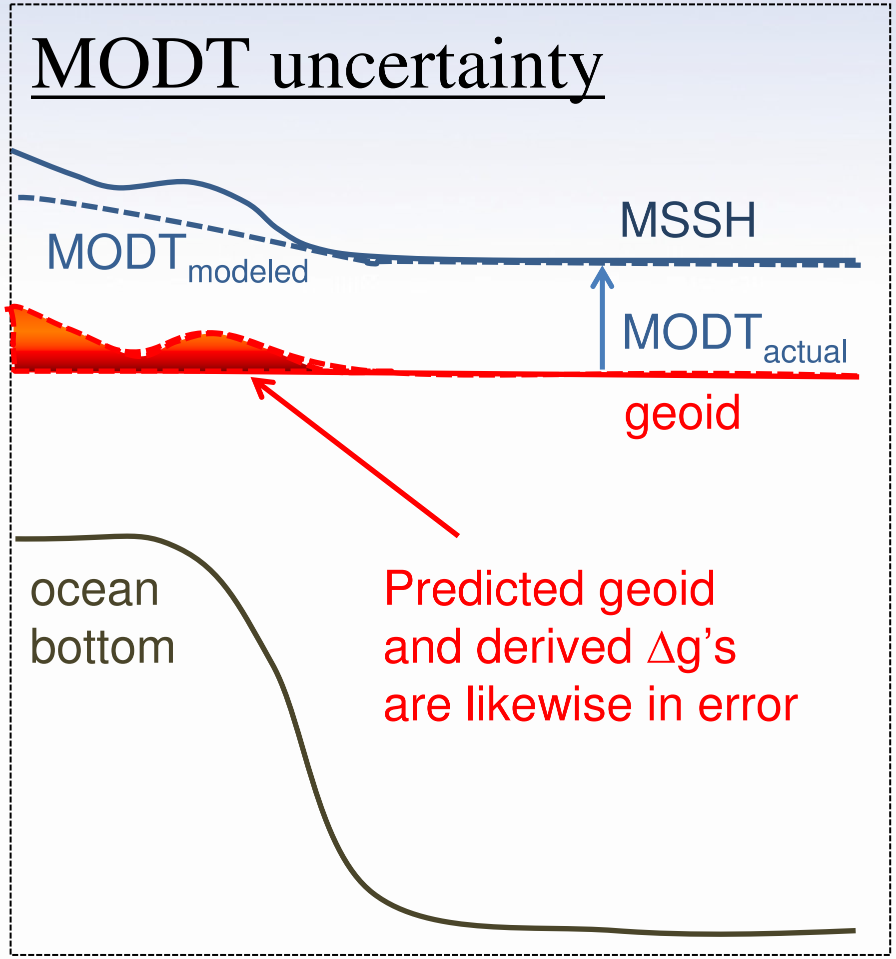


Altimetric Anomalies

- Missions
  - GEOSAT
  - ERS-1/2
  - TOPEX/Poseidon
  - Jason
  - ICESAT
  - ENVISAT
  - ALTIKA
- Considerations
  - Radar signal/footprint
  - Orbit uncertainties
  - Maximum latitude
  - Track spacing (orbit period)
  - Ice or water returns
- Danish Products
  - DTU10
  - DNSC08
- Other Products
  - SIO
  - GSFC00

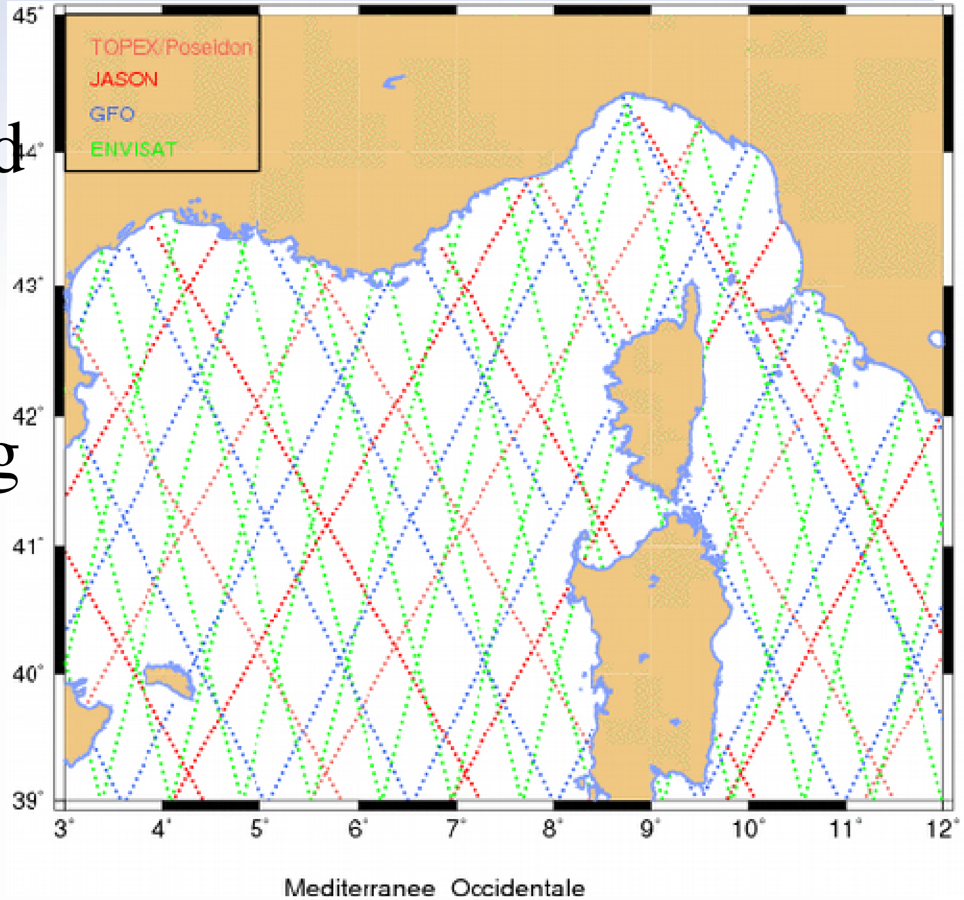
"Gravity" from an Altimeter

- Biggest assumption is that the MSSH can be used to estimate Δg's
- MSSH = geoid + MODT
- MODT valid deep ocean
- Littoral regions poorly known (depths < 500m)
- Re-tracking may help MSSH but not MODT

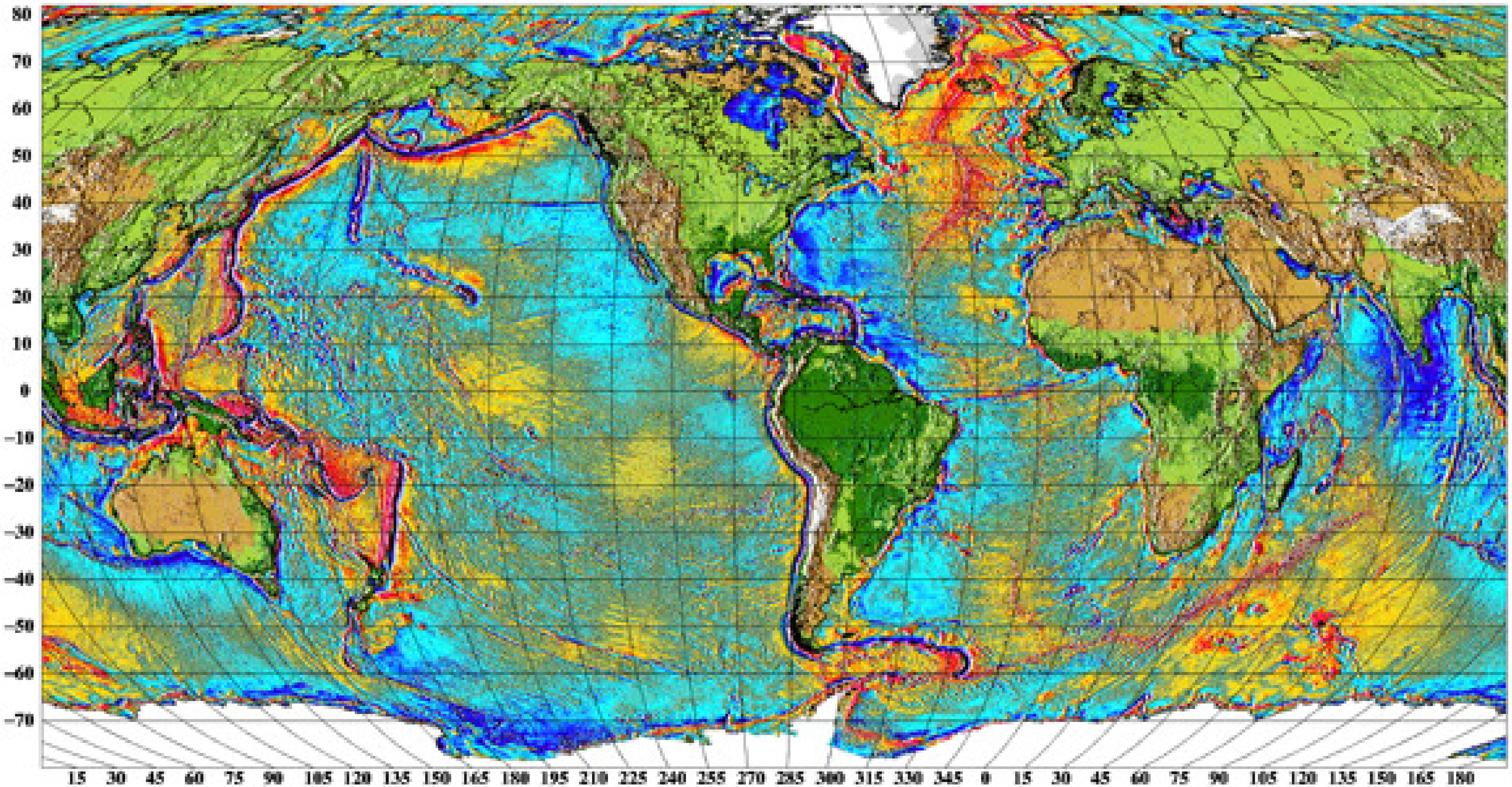


Altimeter Profiles

- Image from Centre of Topography of the Oceans and the Hydrosphere (CTOH)
- http://ctoh.legos.obs-mip.fr/
- Note disparity in track spacing
- Satellite orbits vary a lot
- SRTM only covered ± 60
- Most are sun-synchronous
  - 117 degree inclination => ±83



Free Air Gravity Anomalies from Satellite Altimetry



Andersen, O. B., P. Knudsen and P. Berry (2010) The DNSC08GRA global marine gravity field from double retracked satellite altimetry, Journal of Geodesy, Volume 84, Number 3, DOI: 10.1007/s00190-009-0355-9.

Aerogravity: Bridging the gap between Satellites and Ground

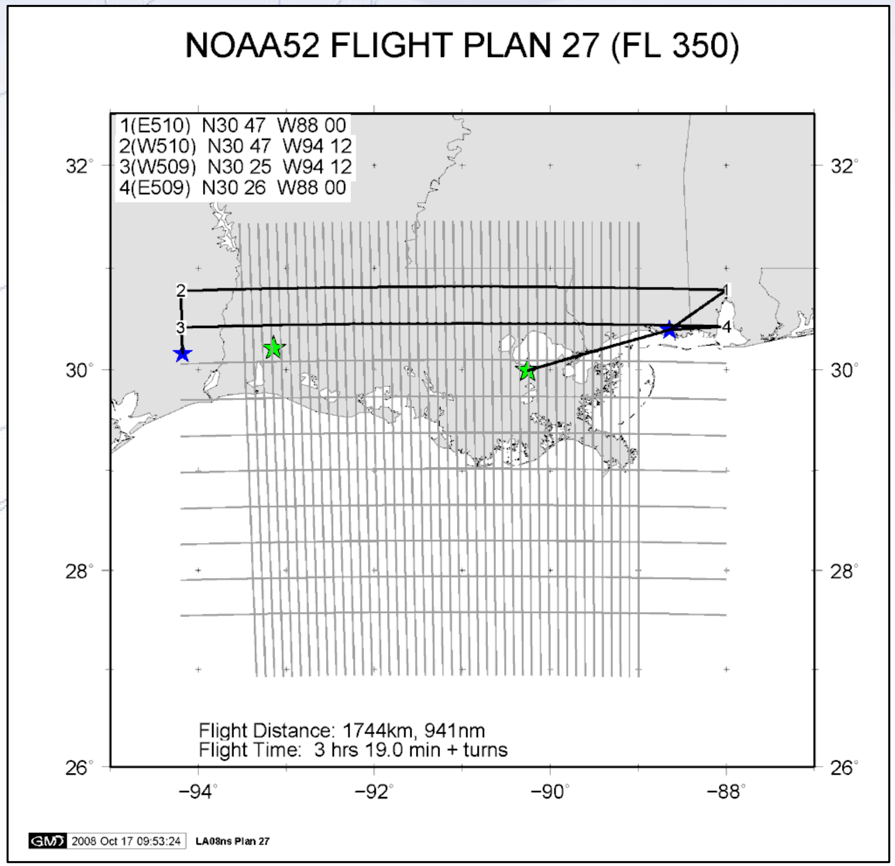
- Satellite models provide long wavelength
  - 300 km and larger scales
  - Provide basis for unification
- Surface gravity data
  - Each observation contains total signal of Earth
  - Extent of gravity surveys covers maximum signal
- Aerogravity spans the other two
  - Determined by extent of survey (e.g., 20-500 km)

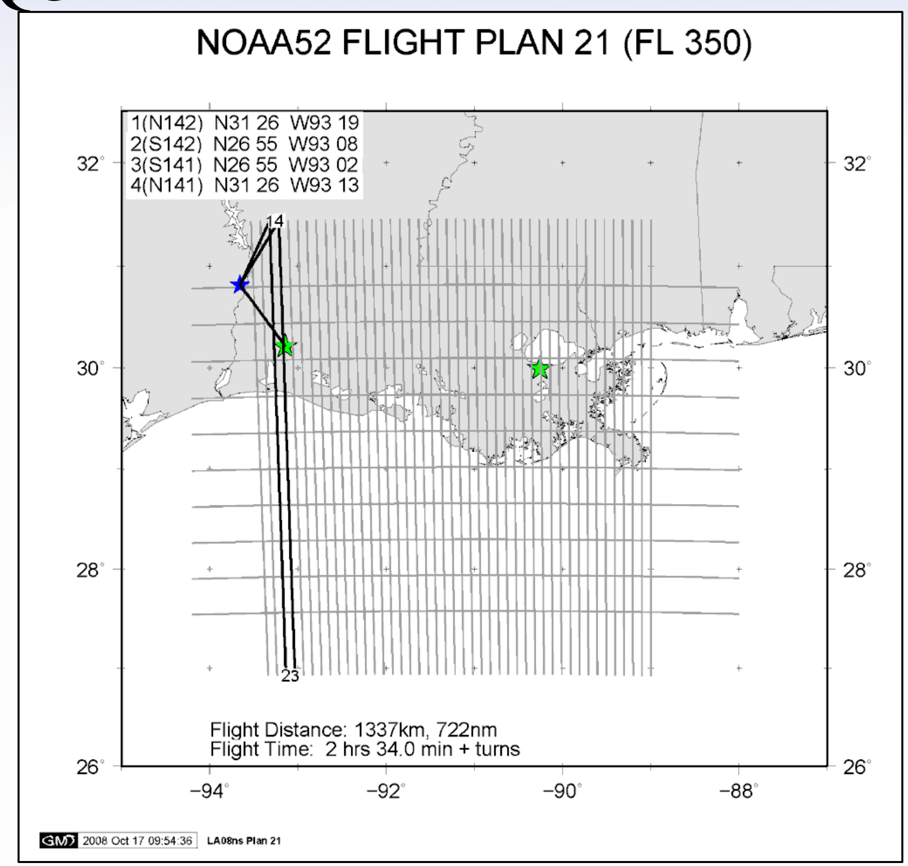
Other Benefits of Aerogravity

- Can span a greater territory than surface campaigns
- Can range over mountains and waterways easily
- Can connect different observational environments
  - terrestrial to shipborne to altimetric
- Can be fixed to satellite modeled gravity at long λ
- Can fix systematic errors in surface surveys (bias)
- Equipment, techniques, software, and procedures are already established and being refined

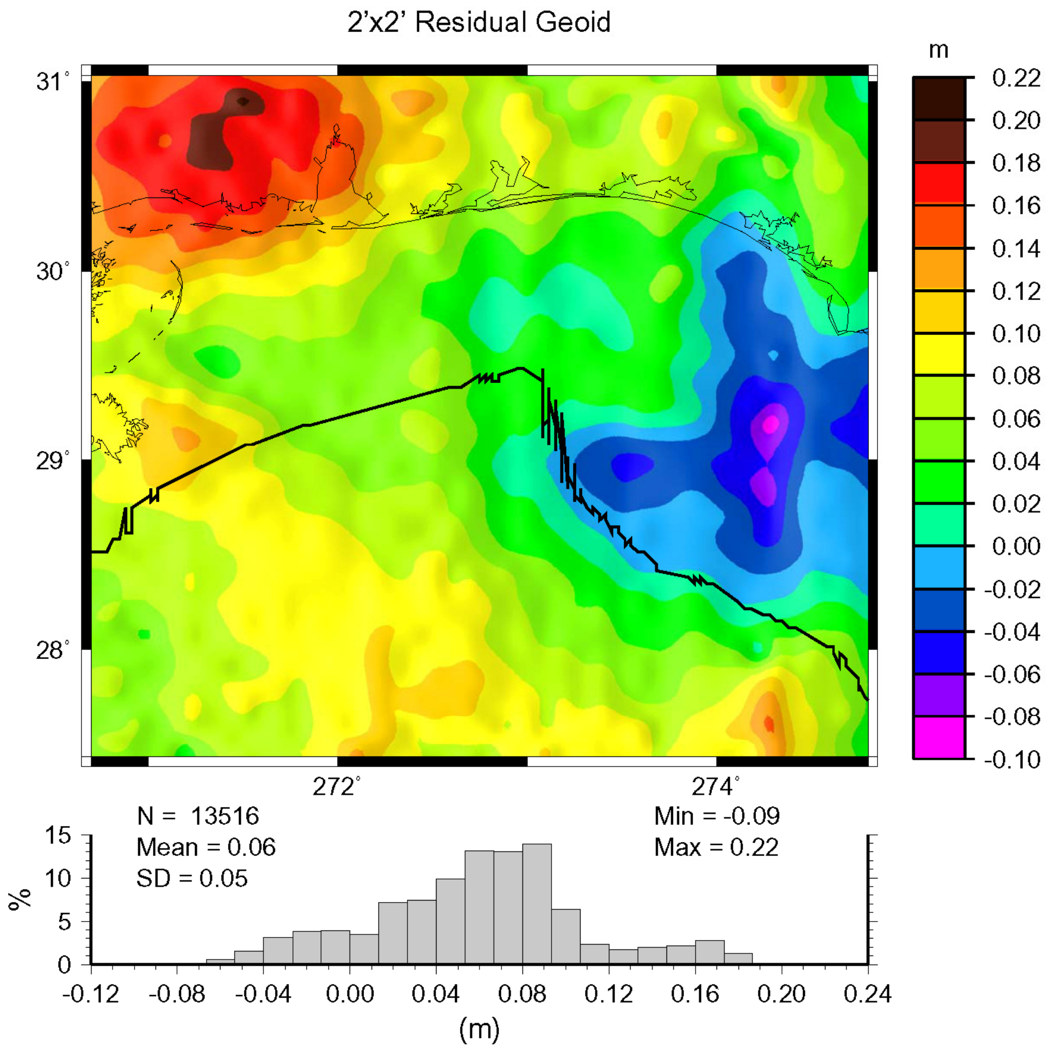
Flight Plans

- Data lines are 10 km apart
- Cross-tracks at 40-50 km for QC

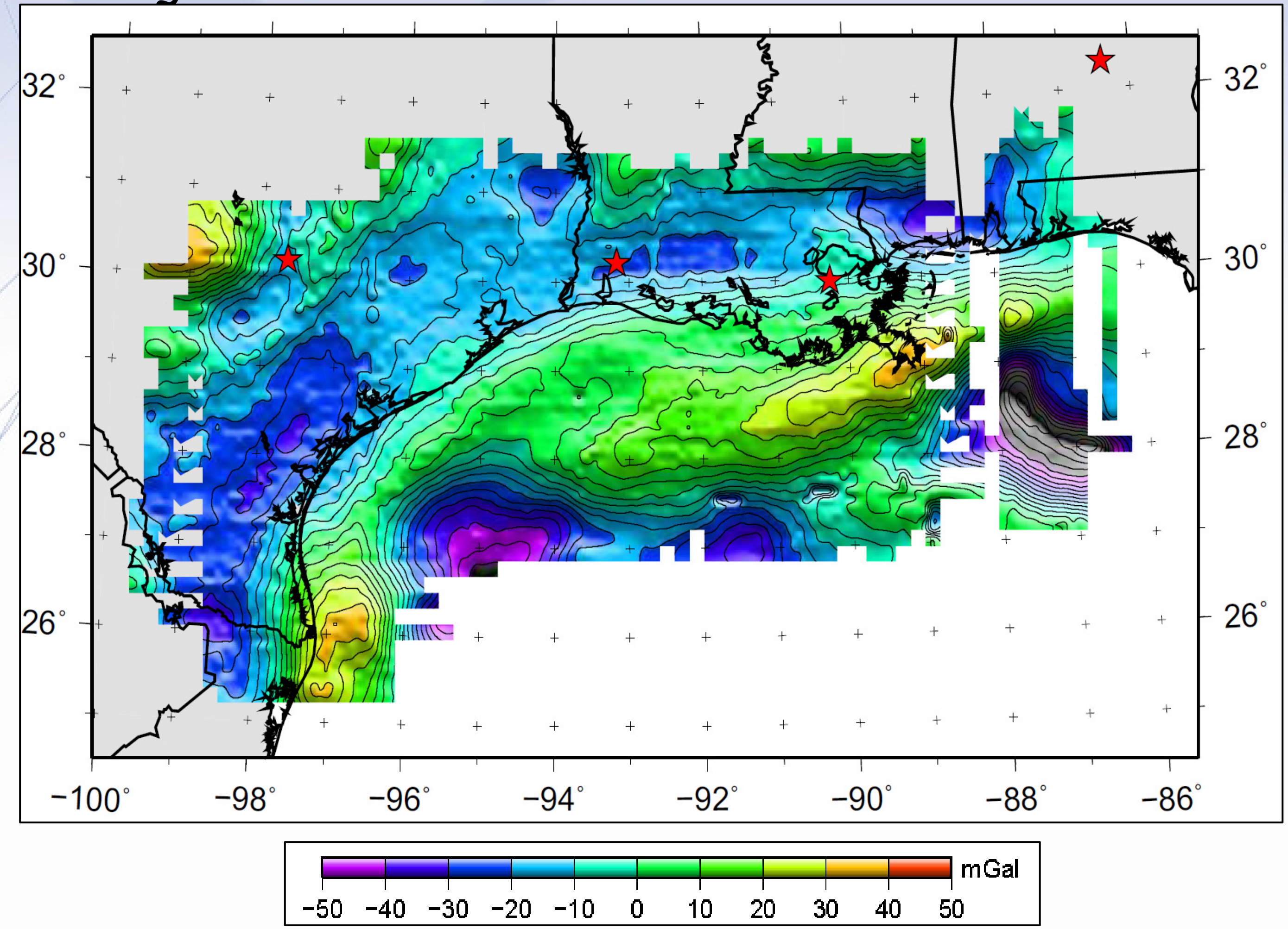




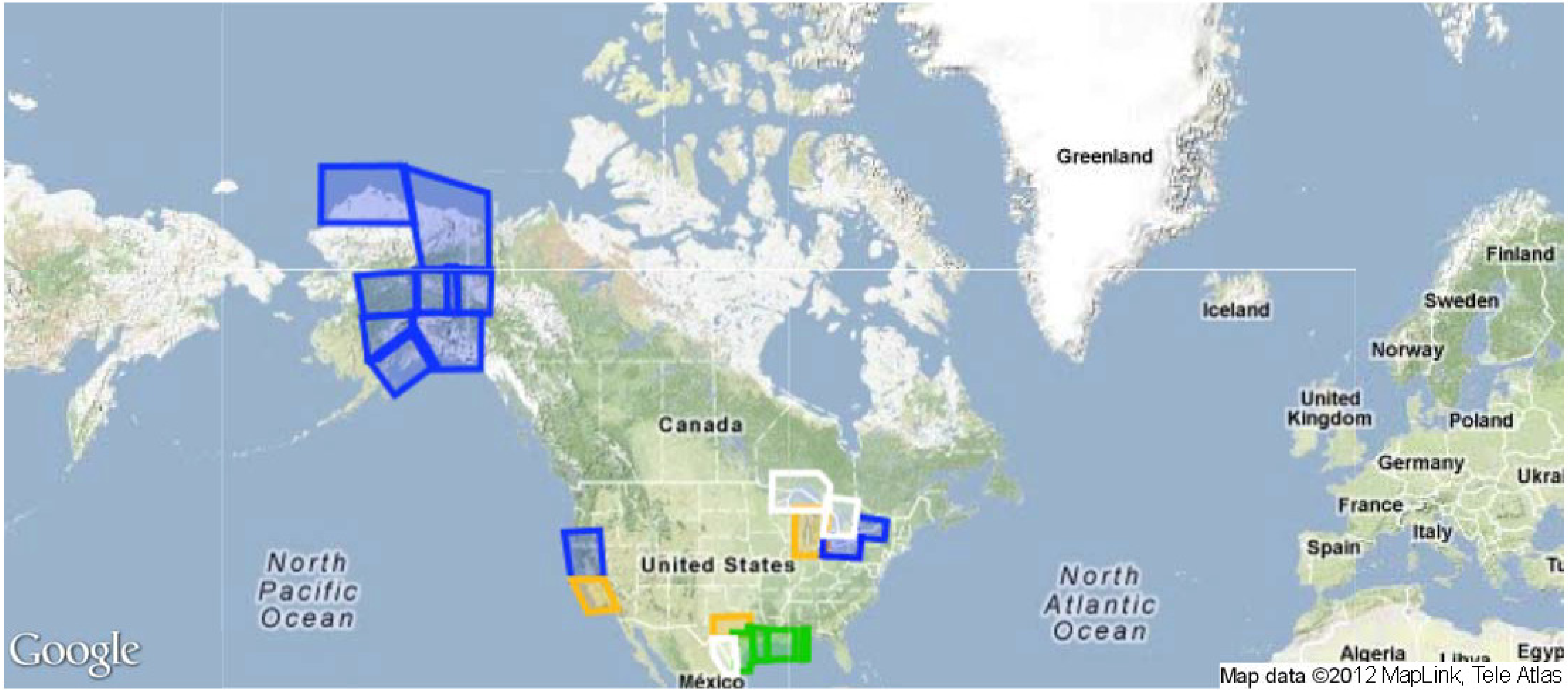
Implied Geoid Changes for GLS06



Survey for the western Gulf Coast



GRAV-D Coverage to date

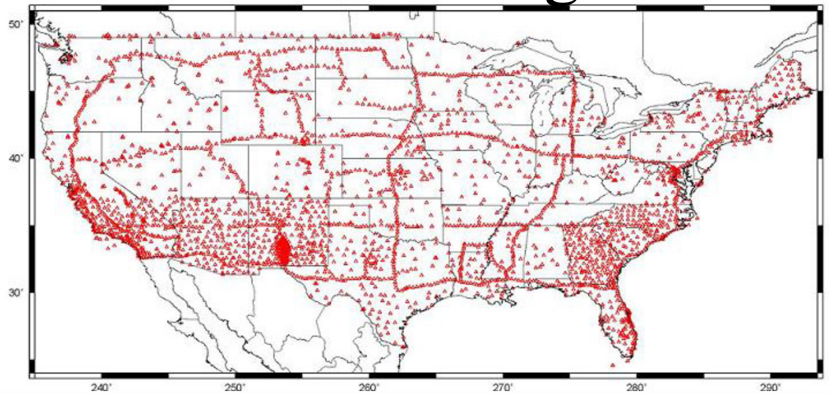


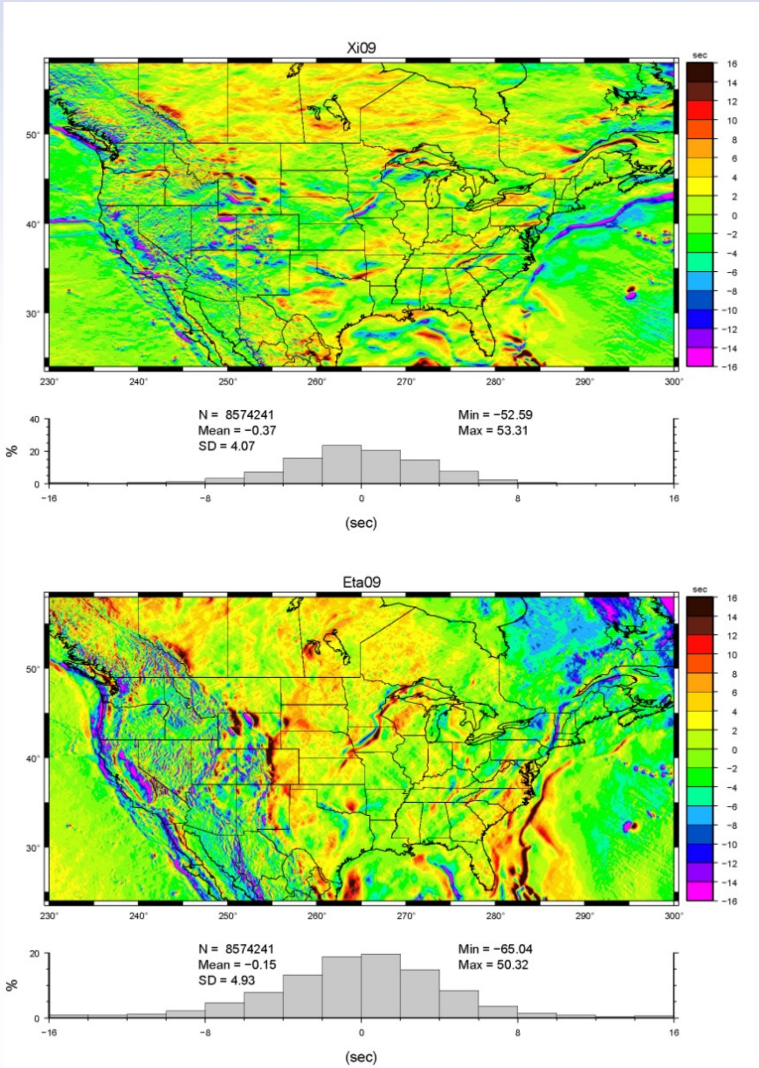
Green: Available data and metadata
Blue: Data being processed, metadata may be available

White: Planned for data collection
Orange: Data collection underway

Deflection of the Vertical (DoV's)

- Another aspect of gravity field
- Angular difference of normals to ellipsoid and geoid
- Measured N-S (ξ) & E-W (η)
- Can help reduce drift in IMU's
- Determined from geoid model

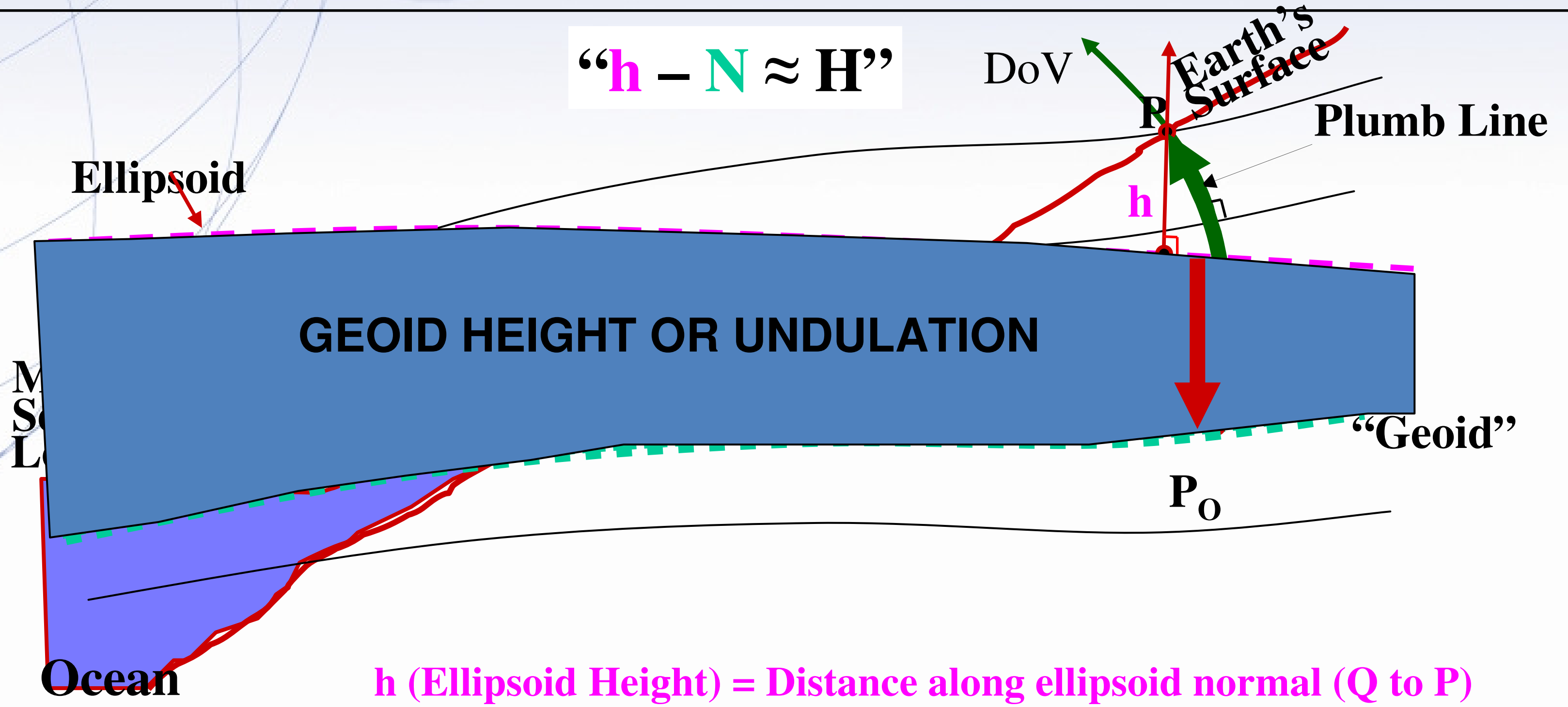




Point Estimates of Geoid Height: GNSS/Leveling

- This is, ultimately, the desired product
- However, these data will be sparse & irregular
- Care must be taken if these are combined in a single least squares adjustment with other functionals of the gravity field (e.g., Δg, ξ/η)
- Systematic effects must be guarded against
  - Is the vertical datum being used actually a geopotential surface?

Relationship between ellipsoid, geoid and orthometric heights.



h (Ellipsoid Height) = Distance along ellipsoid normal (Q to P)

N (Geoid Height) = Distance along ellipsoid normal (Q to P_O)

H (Orthometric Height) = Distance along plumb line (P_O to P)

Earth Gravity Models (EGM's)

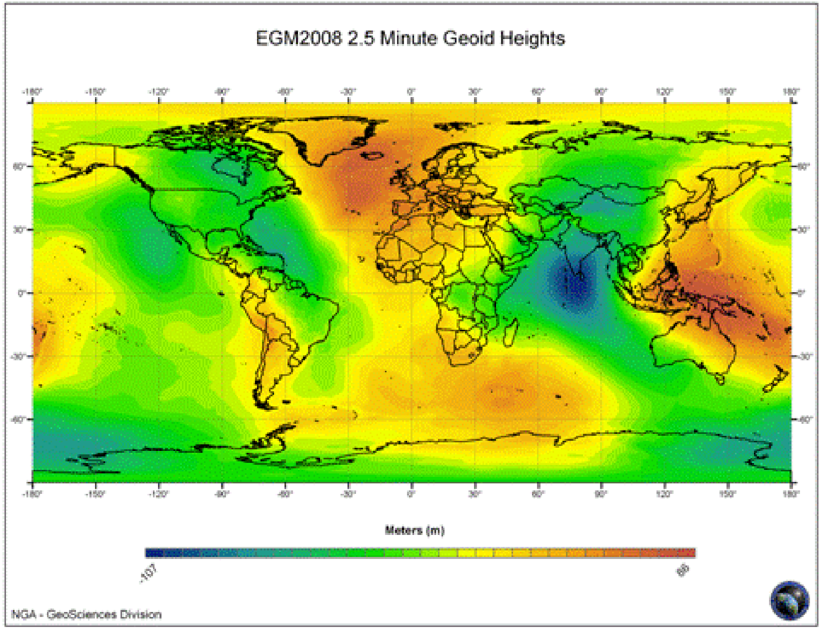
- How to combine all these different data types?
- EGM's have accomplished this mostly:
  - EGM1996(EGM96)-produced by NIMA (NGA)
    - Problems in long wavelength (no GRACE/GOCE data)
    - Had to meld terrestrial solution with satellite (d. 72)
    - Good through degree 360 (111 km resolution)
  - EGM2008(EGM08)-produced by NGA
    - Much more refined solutions (incl. GRACE)
    - Good through degree 2160 (11 km resolution)

EGM's (continued)

- Modeling is by inversion of EHM coefficients
- Harmonics are increasingly complex functions
- Applying a weight to these functions increases or decreases their power
- When added, the harmonics match the unique shape of the Earth's gravity field
- The trick then is to solve for the weights with the gravity observations and functions known (LSC)

EGM2008

- geoid undulation values with respect to WGS 84
- Tide-Free System
- NGA developing a WGS-84 ⇔ IGS08 transformation
- Nominally 5' (11 km) model
- Data in some regions is 15'
- Grids are available
- Can also get EHM coeff.



a=6378137.00 m (semi-major axis of ellipsoid)
f=1/298.257223563 (flattening of ellipsoid)
GM=3.986004418 x 10^{14} m^3s^{-2} (Product of the Earth's mass & Gravitational Constant)
ω=7292115 x 10^{-11} radians/sec (Earth's angular velocity)

http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html

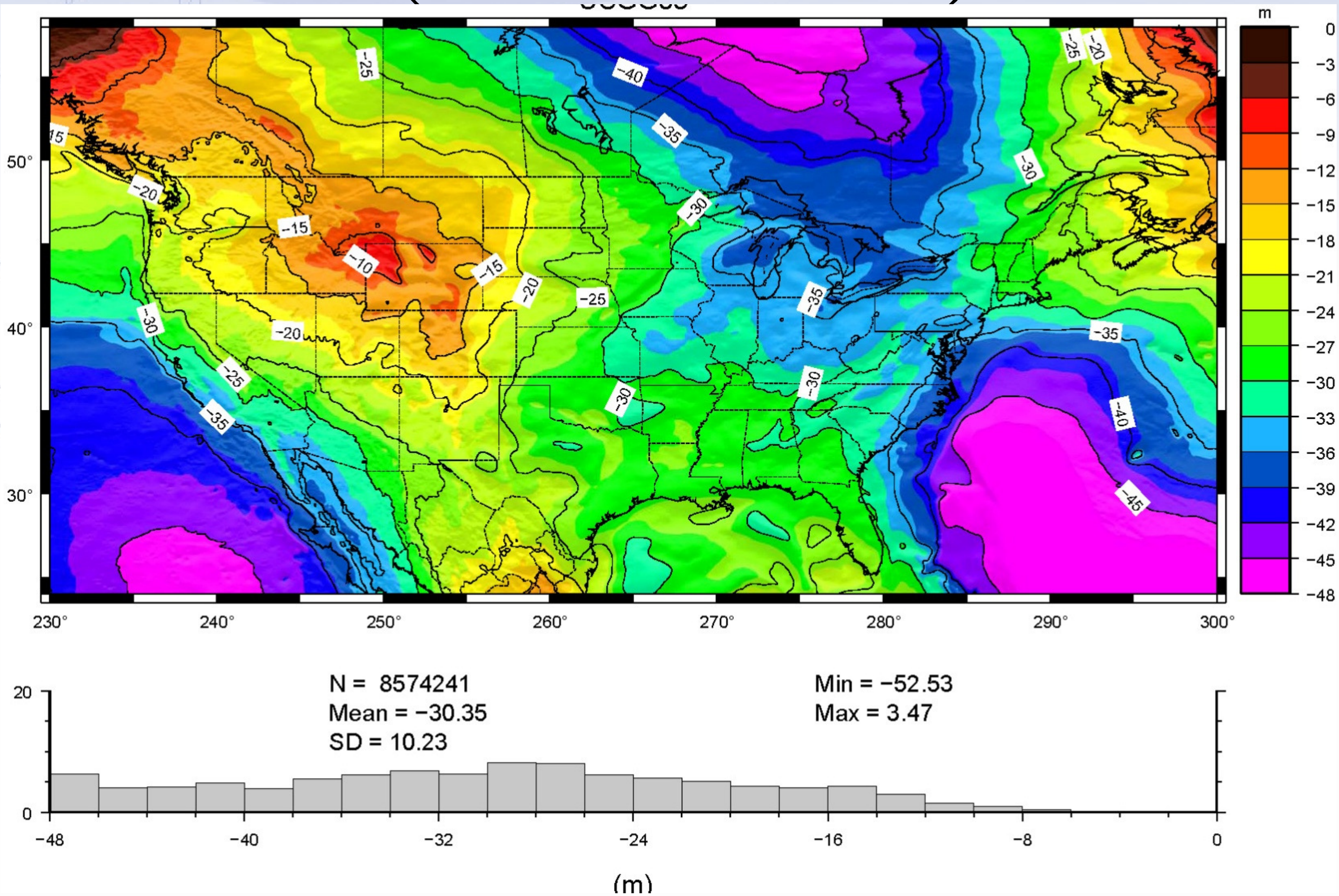
Regional Modeling

- General approach is to start from an EGM and refine using Remove-Compute-Restore Technique
- Gravity observations from multiple sources: terrestrial, shipborne, airborne, & altimetric
- NOTE: A lot of these same data went into EGM2008
- Must adopt an approach: Helmert vs. Molodensky
- Must account for terrain using RTM or TC
- Yields localized gravimetric geoid height model
- Does this match your vertical datum though?

Developing Regional Models

Molodensky	Helmert
Height anomalies	Geoid heights
Telluroid follows terrain	Geoid approximates MSL
Surface gravity anomalies	"Downward" continued to MSL
No assumptions about ρ	Must assume ρ to geoid
Russian approach from 60's	Traditional approach
Much more rigorous theory	Lots of assumptions required
Inversion is very rough	More rigor with adv. computing
Popular in flatter countries	More consistent across mountains
Normal heights	Helmert orthometric heights

U.S. Gravimetric Geoid for 2009 (USGG2009)



World Height System vs. National Vertical Datums

- Recent efforts to realize a WHS have centered on use of EGM2008 or at least GRACE/GOCE
- However, this isn't acceptable to most National Cadastral Agencies with legacy Vertical Datums
- Movement is to adopt gravimetric geoid height models as basis for updated national vertical datums
- This would greatly facilitate development of a WHS since gravimetric geoids will likely be based on EGM2008 or GRACE/GOCE

Using a Geoid model as a National Vertical Datum

- This is easy – iff the national level datum is actually consistent with geopotential surfaces
- Then the geoid model will be consistent, although a bias might be present
- What if the national level datum has a bias and a tilt (i.e., isn't an equipotential surface)?

North American Vertical Datum 1988 (NAVD 88)

- Defined by one height (Father Point/Rimouski)
- Water-level transfers connect leveling across Great Lakes
- Adjustment performed in Geopotential Numbers
- Helmert Orthometric Heights:
  - $H = C / (g + 0.0424\ H_0)$
  - C = geopotential number
  - g = surface gravity measurement (mgals)
  - H_0 = approximate orthometric height (km)
- H = 0 level is nearly a level surface
- H = 0 level is biased and **tilted** relative to best available satellite-based geoid models

Vertical Control Network NAVD 88

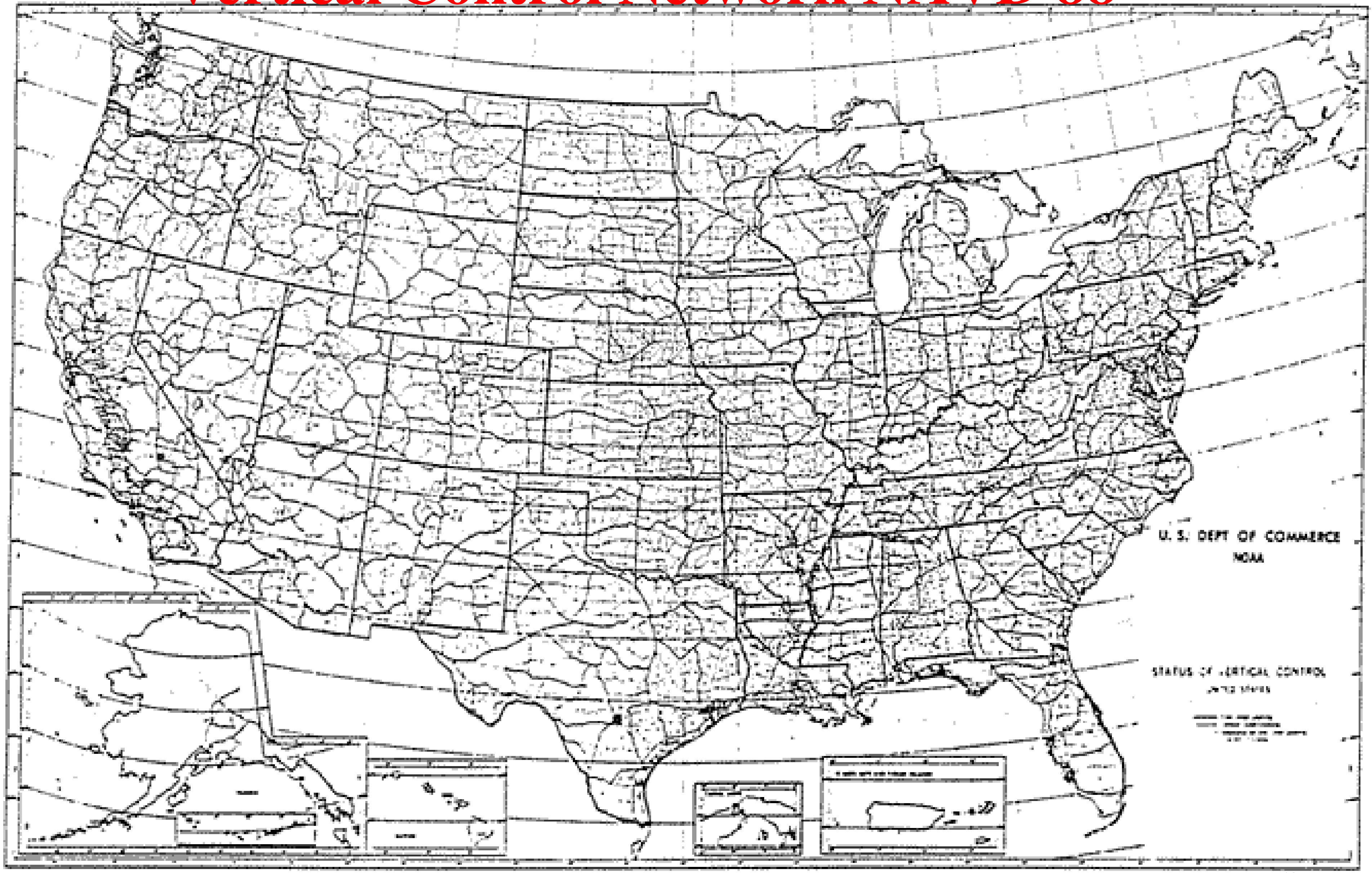
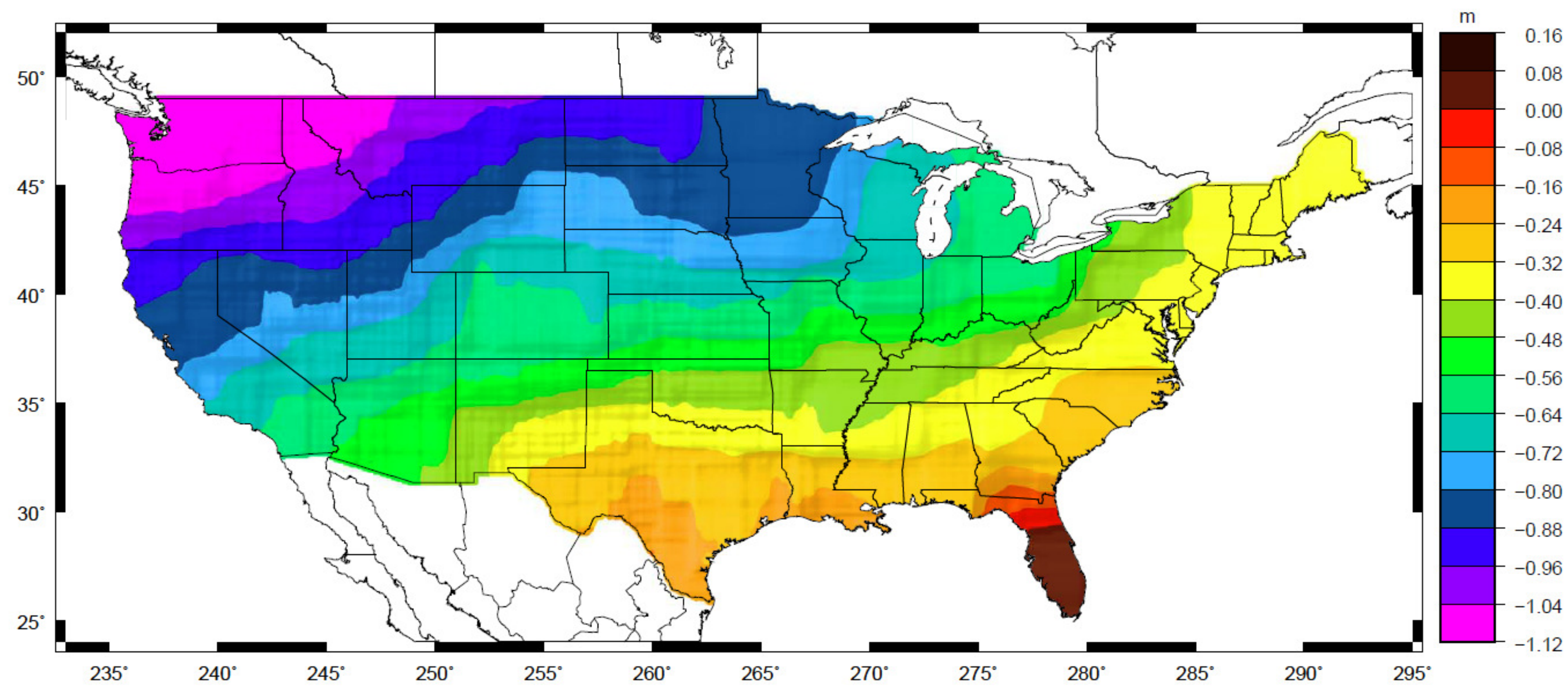


Figure 3. Vertical control used in 1988 adjustment.

450,000 BM's over 1,001,500 km

Why isn't NAVD 88 good enough anymore?

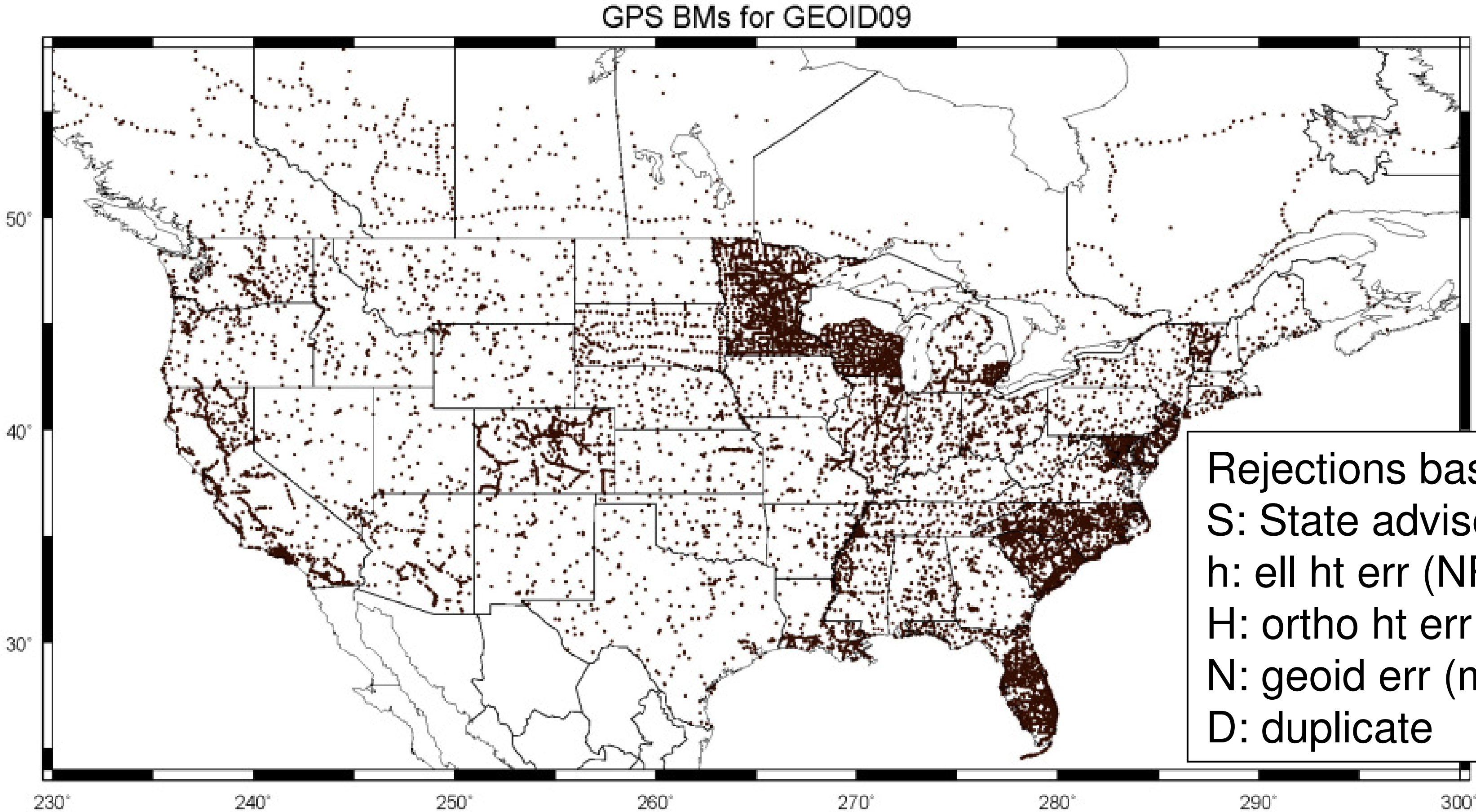


Approximate level of error known to exist in the NAVD 88 zero elevation surface

Hybrid Modeling Using Control Data and a Geoid

- Assumption is that the GNSS/leveling implied geoid height differs from the gravimetric geoid
    - $h - H \neq N$
- To a large extent, the above is never true
- If significant systematic biases exist, an alternative approach is required
- After creating the gravimetric geoid separately, it is warped to fit the level datum control data

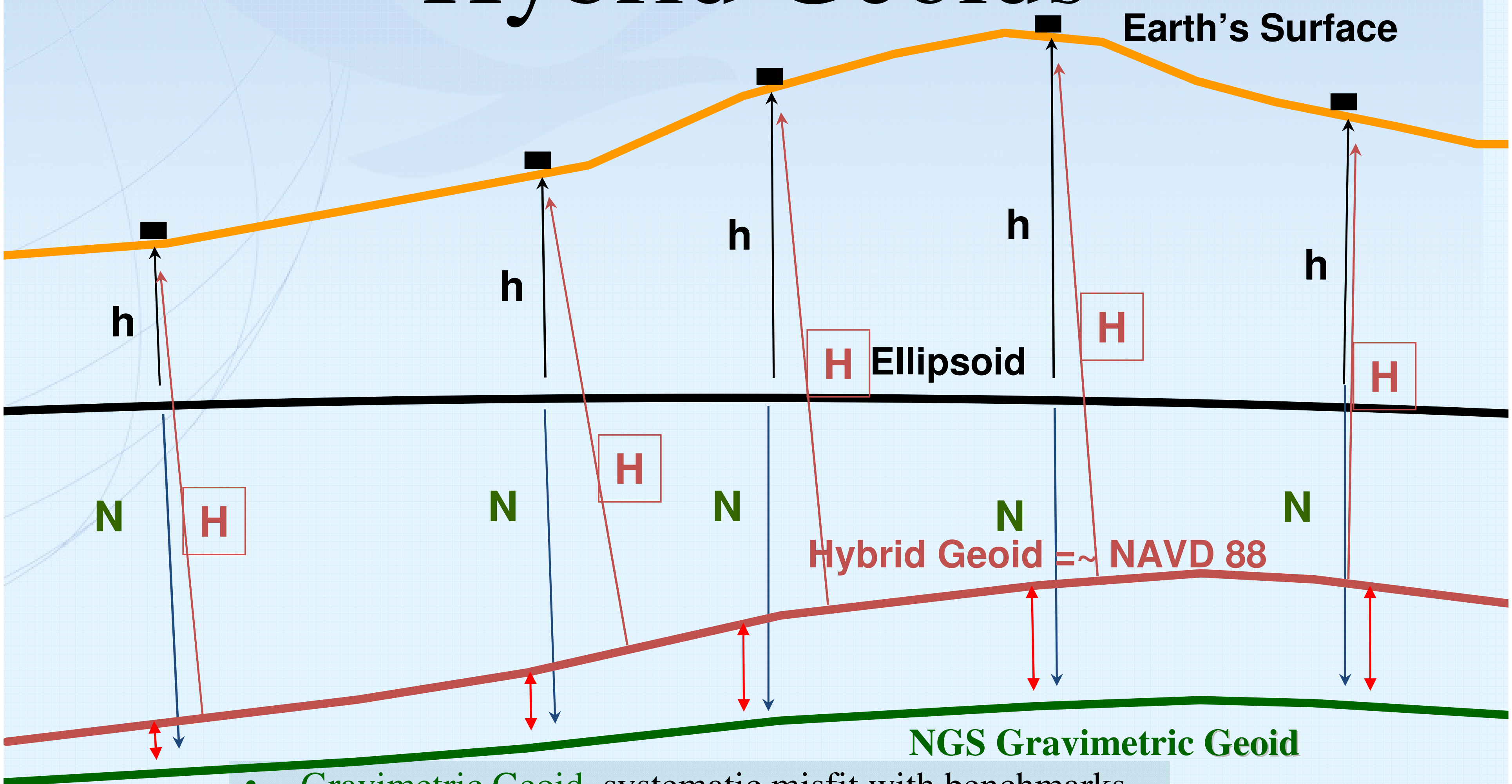
GPSBM2009 (GEOID09 Control Data)



Rejections based on:
S: State adviser
h: ell ht err (NRA)
H: ortho ht err
N: geoid err (misfit)
D: duplicate

20446 total less **1003 rejected** leaves 18,867 (CONUS) plus 576 (Canada)

Hybrid Geoids



- Gravimetric Geoid systematic misfit with benchmarks
- Hybrid Geoid biased to fit local benchmarks
- e = h – H - N

Official U.S. Datums

REGION	CONUS	Alaska	American Samoa	Guam	CNMI	Puerto Rico	US Virgin Islands
Ellipsoidal Reference Frame	NAD83 (NSRS 2007)	NAD83 (CORS96)	NAD83 (PACP00)	NAD83 (MARP00)	NAD83 (MARP00)	NAD83 (CORS96)	NAD83 (CORS96)
Vertical Datum	NAVD88	NAVD88	ASVD02	GUVD04	NMVD03	PRVD02	VIVD09

NAVD88 - NORTH AMERICAN VERTICAL DATUM OF 1988
ASVD02 - AMERICAN SAMOA VERTICAL DATUM OF 2002
GUVD04 - GUAM VERTICAL DATUM OF 2004
NMVD03 - NORTHERN MARIANAS VERTICAL DATUM OF 2003
PRVD02 – PUERTO RICO VERTICAL DATUM OF 2002
VIVD09 – VIRGIN ISLANDS VERTICAL DATUM OF 2009 –pending adoption

Future U.S. Vertical Datum

- U.S. will adopt a gravimetric geoid in 2022 (USGG2022 => GEOID22)
- GRAV-D combines GRACE, GOCE, aerogravity, terrestrial data, DEM, & DDM into a geoid model
- Already selected $W_0 = 62{,}636{,}856.00\ m^2/s^2$
- Canada will be adopting such a model in 2013
- U.S. and Canada will work to adopt a common North American Geoid and use for IGLD 15
- Use of GRACE/GOCE ties U.S. model to WHS

Summary

- Gravity, geopotential, DoV's, and heights are all related functions of the Earth's gravity field
- In many instances, obtaining one type of data is sufficient for resolving all the others
- Hence, collecting gravity from space or in a plane can help determine heights on the ground
- Long wavelengths are best resolved by satellites
- This provides global consistency for all models

Summary (cont.)

- GRACE and GOCE will likely resolve gravity field to degree 200 about 200 km scale features
- Terrestrial gravity can have systematic features that create dm- to m-level errors in the geoid
- The effects of the terrain must also be taken into account or risk making data look "noisy"
- Aerogravity provides means to bridge the gap between surface gravity and satellites gravity

Summary (cont.)

- Aerogravity can span across different environments (land and ocean)
- It can provide a seamless connection through gravity field across a continent and through middle of gravity spectrum
- DoV's can be derived from other observed data and yield better control for IMU's
- Earth Gravity Models, such as EGM2008, use all available data for a comprehensive reference model

Summary (cont.)

- Adoption of gravimetric geoids is the best means of improving accuracy of National Vertical Datums
- New Zealand has already done so; Canada will do so in 2013; the U.S. will do so in 2022
- Using a globally accepted reference field will best ensure a tie-in to a World Height System
- Regional geoid modeling will provide local refinements with a goal of cm-level accuracy

Summary (cont.)

- Regional models can be built from EGM's
- These models provide basis for comparison to National Vertical Datums
- Involved procedures may be required to fit regional gravimetric geoids through control data
- This hybrid process keeps the character of the geoid between points while fitting to the Datum
- Systematic datum errors are preserved though

Thank you for your attention

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