An Introduction to Global Navigation Satellite Systems

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Navigation Satellite Concept and History
Navigation

- Navigation is the process of determining position and direction.
- Generalization of the problem: estimate unknown parameters based on related observations
  \[ z = h(\theta) + v \]

  - \( \theta \) = parameter vector (e.g., Cartesian position and velocity, our “state”)
  - \( Z \) = observation vector (i.e., set of measurements)
  - \( V \) = observation noise vector (i.e., measurement error)
  - \( h(\cdot) \) = relation between parameter set and observation set (i.e., measurement model)
- Given a parameter set, we seek an observation set, a relation between our parameters and observations, and an estimator \( \hat{\theta}(\cdot) \), in order to form an estimate: \( \hat{\theta} = \hat{\theta}(z) \)
- Elegant and effective solutions have been devised by humans and other species for millennia.
Navigation (continued)

- **Relative navigation**
  
  Dead reckoning: monitor rate of travel and heading using a compass; prone to error, especially at sea

  Landmark bearings: angles to two known landmarks will constrain position in two dimensions

- **Absolute navigation: latitude and longitude (clocks vs. celestial)**

  Latitude: Measure the elevation of pole star above the horizon with a sextant or astrolabe

  Longitude: Very good clock or celestial (sextant for the elevation of celestial bodies above the horizon, accurate clock to determine the time of observations, almanac to find the predicted position of the body, magnetic compass to determine azimuth and maintain course continuity between celestial observations)
Radionavigation

- Measurements: *distances* from known transmitter locations via the measurement of radio frequency signal transit time

- Solution to the estimation problem: *trilateration*, the determination of absolute or relative locations of points by measurement of distances using the geometry of circles, spheres, or triangles

- Ground based:
  - LORAN (1940s), Omega (1960s)

- Satellite-based:
  - Sputnik I (1957), Parus and Tskikada, Transit, MOSAIC, and SECOR (1960s)
GNSS

• Global Navigation Satellite Systems (GNSS): radionavigation perfected

• Features

  Accuracy: 3D accuracies of a few meters and down to millimeters for users with specialized equipment and processing

  Availability: signal availability anywhere on Earth with a clear view of the sky

  Integrity: the assurance that expected performance will be realized

[Diagram of Global Positioning System (GPS)]
GNSS (continued)

- **Space segment**
  Constellation of satellites in near-circular, Medium Earth Orbits (~20,000 km) or Geosynchronous Earth Orbits (~36,000 km), each satellite equipped with atomic clocks

- **Control segment**
  Network of ground stations and antennas that perform monitoring of the constellation, check for anomalies, generate new orbit and clock predictions, build and send upload to spacecraft

- **User segment**
  GNSS receivers—specialized radios that track GNSS signals and produce position and velocity solutions, typically with low-cost clocks
GNSS applications

- Applications of satellite navigation are everywhere
- Military
- Civilian
  - Transportation
  - Public services
  - Precise machine control
  - Timing and frequency
  - Surveying
  - Surveillance
  - Recreational Space
  - Scientific applications
- US spends about $1.0B-$1.5B on GPS annually
- Annual direct economic benefit estimated at about $70B, 0.4% GDP*

*www.gmps.gov/governance/advisory/meetings/2015-06/Leveson.pdf
GNSS constellations

- GNSS is an umbrella term for satellite constellations that broadcast signals from space for radionavigation
  
  Systems with global coverage: GPS (United States), Galileo (European Union), GLONASS (Russia), BeiDou (China)
  
  Systems with regional coverage: NAVIC (India), QZSS (Japan)
Approximate GNSS Timeline

GPS Milestones
- Block I
- Block IIR
- Block IIRM
- Block II/IIA
- Block IIF
- Block III

GPS Satellites
- GLONASS
- GLONASS - M
- GLONASS - K1 and K2
- Galileo
- Galileo IOV phase
- Galileo Operational Sats
- Beidou
- Beidou Demonstration Phase (GEO-Regional)
- Beidou II - GEO and MEO
- Beidou/Compass

GLONASS
- 24 Satellites
- Partial Constellation

Galileo
- 2 Demo Satellites
- Initial Operational Capability

Beidou
- Demonstration Phase (GEO-Regional)

Other Systems
- L1C FOC (24 Sats)
- L5 FOC (24 Sats)
- First L5 Signal
- End of Selective Availability
- Codeless Tracking Sunset Date

Timeline:
- 1975: Full Operational Capability (FOC)
- 1980: First L2C Signal
- 1990: L2C FOC (24 Sats)
- 1995: First L5 Signal
- 2000: L5 FOC (24 Sats)
- 2005: First L1C Signal
- 2010: L1C FOC (24 Sats)
- 2015: End of Selective Availability
- 2020: Codeless Tracking Sunset Date
- 2025: Block III
GNSS status and future development

GPS
US
(24+)

Galileo
EU
(27)

GLONASS
Russia
(24)

Beidou
China
(35)
GNSS status and future development

- Status of current GNSS constellations

  **GPS** (US) - Fully operational with global coverage since 1995, 31 satellites in orbit, GPSIII SV01 available for launch and SV01-10, issued a request for proposals in February 2018 for the next block of satellites, GPS IIIF, awarded to Lockheed Martin Sept. 2018

  **GLONASS** (Russia) – Full operational capacity / global coverage achieved in 1995, lost and then regained in 2011, 23 satellites currently operational, modernization efforts underway

  **Galileo** (European Union) – First launch in 2005, full operational capacity expected in 2020 with 27+ satellites, initial services available now with 16 full operational satellites in orbit

  **BeiDou** (China) – currently providing service in Asia-Pacific, expected to reach global coverage in 2020 with 35 satellites, includes MEO, HEO, and GEO sats.

  **NAVIC** – Provides service to India region with GEO/GSO satellites, now operational, 7 satellites

  **QZSS** – Provides service to Japan region, HEO orbits for improved performance in urban canyons, compatible with GPS, operational status in late 2018 with 4 satellites, future expansion planned.

  **SBAS** – Numerous regional Satellite Based Augmentation Systems (SBAS), e.g., WAAS, EGNOS, SDCM, enhance GPS performance and integrity especially for civil aviation

- International collaboration is facilitated through the International Committee on GNSS (ICG) and other forums with the objective of inter-operability among the different constellations
Satellite-based augmentation systems (SBAS)

- **Wide Area Augmentation System (WAAS)**
  Commissioned in 2003 and operated by the U.S. Federal Aviation Administration (FAA), to enable aircraft navigation in the U.S. National Airspace System (NAS)

- **European Geostationary Navigation Overlay System (EGNOS)**
  Three geostationary satellites and a network of ground stations
  Augments the US GPS satellite navigation system in Europe

- **Japan's Multifunction-Transport-Satellite Satellite Augmentation System (MSAS)**
  MSAS for aviation use was commissioned in 2007

- **India's GPS and Geo-Augmented Navigation System (GAGAN)**

- **Russian System of Differential Corrections and Monitoring (SDCM)**

SBAS: Satellite-based GNSS augmentations [23]
Other augmentations

- **Nationwide Differential GPS System (NDGPS):**
  Ground-based augmentation system of ~80 sites operated by the U.S. Coast Guard, Federal Railroad Administration, and Federal Highway Administration, to provide increased accuracy and integrity to U.S. users on land and water.

- **Local Area Augmentation System (LAAS):**
  Augmentation to GPS that focuses its service on the airport area (approximately a 20-30 mile radius)
  Broadcasts correction message via a very high frequency (VHF) radio data link from a ground-based transmitter
  LAAS is a US activity led by the FAA, but other nations are developing their own ground based augmentation system projects

- **NASA Global Differential GPS (GDGPS) System:**
  GDGPS is a commercial high accuracy (~ 10cm) GPS augmentation system, developed by the Jet Propulsion Laboratory (JPL) to support real-time positioning, timing, and orbit determination requirements.
International coordination

- International coordination is critical to ensure compatibility and interoperability.
- US has bilateral agreements or joint statements with all major international GNSS service providers.
- International committee on GNSS (ICG) Established in 2005 under the umbrella of the United Nations to provide forum for discussion.

  Purpose is to promote voluntary cooperation on matters of mutual interest in order to ensure greater compatibility, interoperability, and transparency among GNSS systems.

13th ICG held in Xi’an, China in November 2018.
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GPS Introduction
GPS history

• Developed by the US Department of Defense
  Early GPS program driver was Trident Missile Program (Submarine launched ICBM)
  Satellites carry a nuclear detonation detection payload
• Early satellite navigation systems
  TRANSIT (Doppler positioning)
  Timation (first atomic frequency standards flown in space)
  USAF 621B Program (use of PRN codes for ranging)
• First prototype GPS satellite launched in 1978
• First Block II (Operational) GPS satellite launched 1989
• Full Operational Capability declared in 1994
Current GPS constellation status

- 31 space vehicles currently in operation
  - 0 GPS IIA
  - 12 GPS IIR
  - 7 GPS IIR-M
  - 12 IIF
- several additional satellites in residual status
- Continuously assessing constellation health to determine launch need
- Global GPS civil service performance commitment met continuously since Dec 1993
GPS – three segments
GPS space segment

- Nominal 24 satellite constellation
  Semi-synchronous, circular orbits (~20,200 km/10,900 nautical miles altitude)
  Repeating ground tracks (11 hours 58 minutes)
  Six orbital planes, inclined at 55 degrees, four vehicles per plane
  - Designed for global coverage (at least 4 sats in view)
- Redundant cesium and/or rubidium clocks on board each satellite
- There have been 1-4 replenishment launches per year in recent years
GPS control segment

**SPACE VEHICLE**
Broadcasts the Signal in Space (SIS) PRN codes, L-band carriers, and 50 Hz navigation message (stored in memory)

**MONITOR STATION**
- Sends raw observations to MCS

**SPACE-TO-USER INTERFACE**

**MASTER CONTROL STATION**
- Checks for anomalies
- Computes SIS portion of URE
- Generates new orbit and clock predictions
- Builds new upload and sends to GA

**GROUND ANTENNA**
- Sends new upload to SV

**CONTROL-SPACE INTERFACE**
GPS control segment (cont.)

- The current operational control segment includes a master control station, an alternate master control station, 12 command and control antennas, and 16 monitoring sites.
- Data from Air Force and NGA monitor stations incorporated into Control Segment Kalman filter solution.
GPS user segment

- GPS receivers are specialized “radios” that track GPS signals and produce position and velocity solutions
  - Wide range of cost/sophistication depending on the application
- Signals from 4 or more GPS satellites are required, but 8-10 are typically available at any time
- Low cost civil (SPS) receivers typically track only the L1 C/A signal
- Precise Positioning Service receivers have special keys that allow tracking of the encrypted military codes transmitted on L1 and L2 signals
- High performance civil receivers use special techniques to track the legacy military signals and now track new signals provided by GPS and other GNSS systems

Military Spacecraft
(~$2,000,000)

Consumer/Recreation
(~$100-500)
Legacy GPS signal structure

- **Two L-band carrier frequencies**
  
  \[ L1 = 1575.42 \text{ MHz} \quad L2 = 1227.60 \text{ MHz} \]

- **Two PRN Codes – Uniquely Identify Each Satellite**
  
  - C/A: Coarse Acquisition (Civilian) Code
    - Broadcast only on L1 carrier
    - Available to all users
    - Short code simplifies acquisition
  
  - P(Y): Military Code
    - Encrypted code sequence
    - Available only to *authorized* users on both L1 and L2 carriers
    - Improved performance, jam resistance

- **PRN Codes are modulated with Navigation Message Data**
  
  Provides ephemeris data and clock corrections for the GPS satellites and additional info
GPS modernization

• **Goals**

  System-wide improvements in:
  
  » Accuracy
  » Availability
  » Integrity

  Robustness against interference
  Improved indoor, mobile, and urban use
  Interoperability with other GNSS constellations
  Backward compatibility

• **Achieved through**

  Modernized Space and Ground segments
  New signals
  Improved “CNAV” data message
GPS modernization: new civil signals

- **L2C @ 1227.60 MHz = 120*10.23 MHz (GPS IIR-M 2005)**
  
  Allows ionospheric error removal for civilian users

  Two time-multiplexed PRN codes, one is dataless

- **L5 @ 1176.45MHz = 115*10.23 MHz (GPS IIF 2009)**
  
  Designed for safety of life applications

  In highly protected ARNS band

- **L1C @ 1575.42 MHz = 154*10.23 MHz (GPS III 2018)**
  
  Interoperable with other GNSS systems

  Multiplexed Binary Offset Carrier modulation reduces interference with L1C/A, may allow higher accuracy tracking

- All modulated with improved CNAV or CNAV-2 packetized data message with forward error coding.

  First demonstration conducted in Jun 2013.

  Pre-operational CNAV now continuously broadcast with daily updates.
GPS modernization: spectrum

As of 4/2017:
- L2C broadcast from 19 satellites
- L5 broadcast from 12 satellites
GPS modernization: space segment status

<table>
<thead>
<tr>
<th>BLOCK IIA</th>
<th>BLOCK IIR</th>
<th>BLOCK IIR-M</th>
<th>BLOCK IIF</th>
<th>GPS III/IIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 operational</td>
<td>12 operational</td>
<td>7 operational</td>
<td>12 operational</td>
<td>In production</td>
</tr>
</tbody>
</table>

- **Coarse Acquisition (C/A) code on L1 frequency for civil users**
- **Precise P(Y) code on L1 & L2 frequencies for military users**
- **7.5-year design lifespan**
- **Launched in 1990-1997**
- **Last one decommissioned in 2016**

- **C/A code on L1**
- **P(Y) code on L1 & L2**
- **On-board clock monitoring**
- **7.5-year design lifespan**
- **Launched in 1997-2004**

**Learn more about GPS IIR at AF.MIL**

- **All legacy signals**
- **2nd civil signal on L2 (L2C)**
- **New military M code signals for enhanced jam resistance**
- **Flexible power levels for military signals**
- **7.5-year design lifespan**
- **Launched in 2005-2009**

**Learn more about GPS IIR-M at AF.MIL**

- **All Block IIR-M signals**
- **3rd civil signal on L5 frequency (L5)**
- **Advanced atomic clocks**
- **Improved accuracy, signal strength, and quality**
- **12-year design lifespan**
- **Launched in 2010-2016**

**Learn more about GPS IIF at AF.MIL**

- **All Block IIF signals**
- **4th civil signal on L1 (L1C)**
- **Enhanced signal reliability, accuracy, and integrity**
- **No Selective Availability**
- **15-year design lifespan**
- **IIF: laser reflectors; search & rescue payload**
- **First launch in 2018**

**Learn more about GPS III at AF.MIL**

[https://www.gps.gov/systems/gps/space/#generations](https://www.gps.gov/systems/gps/space/#generations)
GPS modernization: ground segment

- Legacy ground system is known as the Operational Control System (OCS). Several modernization efforts have been implemented or are in progress.
- **L-AII**: Legacy Accuracy Improvement Initiative (completed 2008)
  - Added 10 NGA monitoring sites to bring total to 16
- **AEP**: Architecture Evolution Plan (2007-current)
  - Broad OCS system upgrade. Manages current modernized constellation
- **LADO**: Launch and early orbit, Anomaly resolution, and Disposal Operations (fielded 2007)
  - Handles GPS satellites outside operational constellation
- **OCX**: Next Generation Operational Control Segment (contract awarded 2008, multiphase rollout 2017-2021)
  - Eventually replace OCS, control GPS III
- **Risk Mitigations to late OCX delivery**
  - **COps**: GPS III Contingency Ops program (delivery in 2019)
    - Enable GPS III’s “IIF features” prior to OCX Block 1 (2021)
  - **MCEU**: MCode Early Use - MCEU
    - OCS upgrades to support MCode operational testing

GPS III Status

- Newest block of GPS satellites
  
  First to broadcast common L1C signal
  
  Multiple civil and military signals;
  
  » L1 C/A, L1 P(Y), L1M, L1C,
  
  » L2C, L2 P(Y), L2M,
  
  » L5
  
  Three Rubidium clocks
  
  First launch in ~2018 timeframe

- Lockheed Martin in Denver CO awarded contract for two (SV01/02) development and six operational satellites (SV03-08), with option for two more (SV09-10) exercised in 2016.

- GPS III SV11+ awarded to Lockheed Martin Sept. 2018. Will add:
  
  Laser Retro-reflector Array
  
  Search and Rescue payload

GPS documentation

- System technical docs available on www.gps.gov
- GPS IS-200:
  - Spec. of legacy C/A & P codes and NAV message
  - Rev E and beyond adds L2C and CNAV
- GPS IS-800:
  - Specification of L5, and L5 CNAV
- SPS & PPS Performance standards
  - Defines the guaranteed level of performance in terms of Signal in Space (SIS) accuracy and Constellation design
  - Current system performance surpasses minimum spec and is improving.
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GPS Space Applications
Space Uses of Global Navigation Satellite Systems (GNSS)

- **Real-time On-Board Navigation:** Precision formation flying, rendezvous & docking, station-keeping, Geosynchronous Orbit (GEO) satellite servicing
- **Earth Sciences:** GPS as a measurement for atmospheric and ionospheric sciences, geodesy, and geodynamics
- **Launch Vehicle Range Operations:** Automated launch vehicle flight termination; providing safety net during launch failures & enabling higher cadence launch facility use
- **Attitude Determination:** Some missions, such as the International Space Station (ISS) are equipped to use GPS/GNSS to meet their attitude determination requirements
- **Time Synchronization:** Support precise time-tagging of science observations and synchronization of on-board clocks

*GPS capabilities to support space users will be further improved by pursuing compatibility and interoperability with GNSS*
Main radiometric instrument:
NASA/JPL BlackJack GPS receiver modified to track gravity-sensing crosslinks, and to form star-camera solutions, while producing cm-level POD and 0.1 nanosecond relative time transfer.

The Onera accelerometer was also required to produce accurate gravity maps.
Results from GRACE

GRACE data have significantly improved understanding of: the global water cycle, mass and energy exchange within and between the Earth System components, the changes in ocean mass, the changing dynamics of polar ice caps and large continental aquifers and improved the prospects for assimilation of mass change data into climate models.

Examples of science applications include:

- Sea level change
- Surface and Deep Ocean Currents
- Ocean heat storage
- Polar and continental ice sheet melt
- Earth system mass transport
- Drought and flooding
- Land surface total water storage
- Modeling and assimilation
- Earthquake assessment
<table>
<thead>
<tr>
<th></th>
<th>EM-1</th>
<th>EM-2</th>
<th>SM-1</th>
<th>EM-3</th>
<th>EM-4</th>
<th>EM-5</th>
</tr>
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<tbody>
<tr>
<td>2021</td>
<td>2022</td>
<td>2023</td>
<td>2024</td>
<td>2025</td>
<td>2026</td>
<td></td>
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<tr>
<td>Block 1: ICPS Cargo</td>
<td>Block 1: ICPS Cargo</td>
<td>Block 1B Cargo</td>
<td>Block 1B: EUS Cargo</td>
<td>Block 1B: EUS Cargo</td>
<td>Block 1B: EUS Cargo</td>
<td></td>
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<tr>
<td>4 Crew</td>
<td>4 Crew</td>
<td>Europa Clipper</td>
<td>4 Crew</td>
<td>4 Crew</td>
<td>4 Crew</td>
<td></td>
</tr>
<tr>
<td>Cis-Lunar Space Mission to confirm vehicle performance and operational capability.</td>
<td>First crewed mission, to confirm vehicle performance and operational capability, same profile as EM-1.</td>
<td>First cargo mission configuration.</td>
<td>First Orion Docking to extract Habitat Module from EUS, deliver to Lunar Orbit Platform - Gateway</td>
<td>Deliver Logistics Module to Lunar Gateway</td>
<td>Deliver Airlock Element to Lunar Gateway</td>
<td></td>
</tr>
<tr>
<td>13 CubeSat Payloads</td>
<td>Orion Capsule + Crew</td>
<td>Europa Clipper to Jupiter</td>
<td>LOP-G Habitat Module</td>
<td>LOP-G Logistics Module</td>
<td>LOP-G Airlock Element</td>
<td></td>
</tr>
<tr>
<td>Cis-Lunar Trajectory 11-21 days</td>
<td>Multi-TU Lunar Free Return 8-21 days</td>
<td>Jupiter Direct 2.5 years</td>
<td>Near-Rectilinear Halo Orbit (NRHO) 16-26 days</td>
<td>Near-Rectilinear Halo Orbit (NRHO) 26-42 days</td>
<td>Near-Rectilinear Halo Orbit (NRHO) 26-42 days</td>
<td></td>
</tr>
</tbody>
</table>


National Aeronautics and Space Administration
MISSION FACTS

LAUNCH DATE 2022

ORBIT: Polar low Earth orbit (LEO)

CLIENT: A satellite in LEO owned by the U.S. government

OPERATIONS: Autonomous rendezvous and grasping with telerobotic refueling and relocation

MANAGEMENT: The Space Technology Mission Directorate at NASA Headquarters and the Satellite Servicing Projects Division at NASA’s Goddard Space Flight Center

SERVICING TECHNOLOGIES

1. Autonomous, Real-Time Relative Navigation System
   Sensors, algorithms and processors join forces, allowing Restore-L to rendezvous safely with its client.

2. Servicing Avionics
   In addition to ingesting and crunching sensor data, these elements control Restore-L’s rendezvous and robotic tasks.

3. Dexterous Robotic Arms
   Two nimble, maneuverable arms precisely execute servicing assignments. Software comes included.

4. Advanced Tool Drive And Tools
   Sophisticated, multifunction tools are manufactured to execute each servicing task.

5. Propellant Transfer System
   This system delivers measured amounts of fuel to the client at the right temperature, pressure and rate.
PACE MISSION
PACE will extend and improve NASA’s 20 plus years of global satellite observations of our living ocean, atmospheric aerosols, and clouds and initiate an advanced set of climate-relevant data records. By determining the distribution of phytoplankton, PACE will help assess ocean health. It will also continue key measurements related to air quality and climate.

Science Goals
To extend systematic ocean color, atmospheric aerosol, and cloud data records for Earth system and climate studies. To address new and emerging science questions by detecting a broader range of color wavelengths that will provide new and unprecedented detail.

Key Mission Characteristics
• Hyperspectral ocean color instrument
• Two multi-angle polarimeters
• Launch readiness date: Fall 2022
• 675 km (419 mi) orbital altitude
• Sun-synchronous, polar orbit
• Global coverage every two days
• Managed by Goddard Space Flight Center

(image credit: NASA)
GPS Antenna Characterization Experiment (ACE)

Overview

- GPS L1 C/A signals from GEO are available at a ground station through a “bent-pipe” architecture
- Map side lobes by inserting advanced, weak-signal tracking GPS receivers at ground station to record observations from GEO

Data Collection & Visualization

- Trace path of GEO vehicle in antenna frame of each GPS vehicle
- Reconstruct full gain pattern after months of tracking
ACE Results Average Transmit Gain – Block IIR

- In-flight averaged over all SVNs in block in 1 deg x 1 deg bins
- Remarkable similarity between average flight and ground measurements
  - Note matching patterns in nulls around outer edge
ACE Results Average Transmit Gain – Block IIA/IIF

- Averaged over all SVNs in block in 1 deg x 1 deg bins
- IIF side lobes are shifted 45 deg in azimuth from other blocks

![Reconstructed Transmitter Gain Pattern Averaged for Block IIA](image1)

![Reconstructed Transmitter Gain Pattern Averaged for Block IIF](image2)
GPS ACE Conclusions

• GPS ACE architecture permits tracking of extremely weak signals over long duration
  – MGPSR produces signal measurements well into back lobes of GPS vehicles
  – 24/7 GPS telemetry provides near continuous tracking of each PRN
• First reconstruction of full GPS gain patterns from flight observations
  – Block averages of IIR, IIR-M show remarkable consistency with ground patterns
    • Demonstrates value in extensive ground testing of antenna panel
  – Characterized full gain patterns from Blocks IIA, IIF for the first time
  – Patterns permit more accurate simulations of GPS signal availability for future HEO missions
• Additional analysis of pseudorange deviations indicate usable measurements far into side lobes
• Dataset available at: https://esc.gsfc.nasa.gov/navigation
Recent Results: Lunar GPS

- NASA has recently published two studies looking at the feasibility of GPS navigation at lunar distances:
  - ION GNSS+ 2017: Winternitz, et al\(^1\)
    - Published MMS Phase 2 results using GPS to 25 RE
    - Projected MMS performance to lunar distance
  - AAS GN&C 2018: Ashman, et al\(^2\)
    - Looked broadly at GPS visibility for different antennas and C/N0 receiver threshold values
    - Validated results vs. MMS and GOES-16 flight data

- These studies represent early GPS-only analyses that could be used as basis for WG-B in-depth analysis.

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NASA’s Magnetospheric MultiScale (MMS) Mission

• Discover the fundamental plasma physics process of reconnection in the Earth’s magnetosphere.
• Coordinated measurements from tetrahedral formation of four spacecraft with scale sizes from 400km to 10km
• Flying in multiple highly-elliptical orbits:
  – Phase 1  1.2x12 R_E (magnetopause) Mar ‘14-Feb ‘17
  – Phase 2B 1.2x25 R_E (magnetotail) May ‘17-present
  – 2019: Apogee raise to 1.2x29 RE
2017 MMS study:
Concept Lunar mission

- Study: How will MMS receiver perform if used on a conceptual Lunar mission with 14dBi high-gain antenna?
- GPS measurements simulated & processed using NASA GEONS filter.
- Visibility similar to MMS Phase 2B, as high-gain makes up for additional path loss
  - Avg visibility: ~3 SVs; C/N0 peaks > 40dB-Hz (main lobes) or > 30 dB-Hz (side lobes)
- Range/clock-bias errors dominate – order of 1-2 km; lateral errors 100-200 m
  - With atomic clock, or, e.g., periodic 2-way range/Doppler, could reduce range errors to meas. noise level

*Top: Signals tracked and radial dist to Earth (red) and Moon (cyan); Bottom: C/N₀*
2018 Lunar GPS Visibility Study

- GPS constellation modeled as accurately as possible, including sidelobe signals; validated with GOES-16 and MMS flight data.

- Calibrated models applied to outbound lunar near-rectilinear halo orbit (NRHO) GPS receiver reception with 22 dB-Hz acq/trk threshold.

<table>
<thead>
<tr>
<th>Peak Antenna Gain</th>
<th>1+ Vis.</th>
<th>4+ Vis.</th>
<th>Maximum Outage</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 dB</td>
<td>63%</td>
<td>8%</td>
<td>140 min</td>
</tr>
<tr>
<td>10 dB</td>
<td>82%</td>
<td>17%</td>
<td>84 min</td>
</tr>
<tr>
<td>14 dB</td>
<td>99%</td>
<td>65%</td>
<td>11 min</td>
</tr>
</tbody>
</table>

- A modest amount of additional antenna gain or receiver sensitivity increases coverage significantly.

National Aeronautics and Space Administration
Conclusions

• These results show useful onboard GPS navigation at lunar distances is achievable now using currently-available signals and flight-proven receiver technology.

• A modest increase in gain or receiver sensitivity increases visibility significantly.

• Future work must extend these specific studies to full navigation analysis of cis-lunar spacecraft, including effects of DOP, and utilizing the full capability of multi-GNSS signals.

• ICG WG-B is a natural forum for these discussions and analyses, in keeping with the ICG-12 recommendation for analysis for cis-lunar missions and beyond.
Potential Future Application: Lunar Orbital Platform—Gateway

- NASA Exploration Campaign: Next step is deployment and operations of US-led Lunar Orbital Platform—Gateway (previously known as Deep Space Gateway)
- Step-off point for human cis-lunar operations, lunar surface access, missions to Mars
- Features include:
  - Power and propulsion element (PPE) – targeted for 2022
  - Human habitation capability
  - Docking/rendezvous capability
  - Extended uncrewed operations (not continuously crewed)
  - Lunar near-rectilinear halo orbit (NRHO)
- Gateway conceptual studies are continuing with ISS partners
  - Requirements to be baselined in 2018
  - To be followed by Broad Agency Announcement for partnerships
- Gateway represents a potential application for on-board GNSS navigation
- NASA will continue providing updates to WG-B as plans develop.

Potential Future Application: Lunar Orbital Platform—Gateway

GATEWAY A spaceport for human and robotic exploration to the Moon and beyond

- **HUMAN ACCESS TO & FROM LUNAR SURFACE**
  Astronaut support and teleoperations of surface assets.

- **U.S. AND INTERNATIONAL CARGO RESUPPLY**
  Expanding the space economy with supplies delivered aboard partner ships that also provide interim spacecraft volume for additional utilization.

- **SIX DAYS TO ORBIT THE MOON**
  The orbit keeps the crew in constant communication with Earth and out of the Moon’s shadow.

- **INTERNATIONAL CREW**
  International crew expeditions for up to 30 days as early as 2024. Longer expeditions as new elements are delivered to the Gateway.

- **SAMPLE RETURN**
  Pristine Moon or Mars samples robotically delivered to the Gateway for safe processing and return to Earth.

- **COMMUNICATIONS RELAY**
  Data transfer for surface and orbital robotic missions and high-rate communications to and from Earth.

- **SCIENCE AND TECH DEMOS**
  Support payloads inside, affixed outside, free-flying nearby, or on the lunar surface. Experiments and investigations continue operating autonomously when crew is not present.

- **ACCESS**
  384,000 km from Earth
  Accessible via NASA’s SLS as well as international and commercial ships.

**GATEWAY SPECS**
- 4 Crew Members
- 30-90 Day Crew Missions
- 125 m³ Pressurized Volume
- Up to 75 mt with Orion docked

**20180813**
Global Exploration Roadmap

- The GER is a human space exploration roadmap developed by 14 space agencies participating in the International Space Exploration Coordination Group (ISECG)

- The non-binding strategic document reflects consensus on expanding human presence into the Solar System, including
  - Sustainability Principles, spaceflight benefits to society
  - Importance of ISS and LEO
  - The Moon: Lunar vicinity and Lunar surface
  - Mars: The Driving Horizon Goal

www.globalspaceexploration.org
www.nasa.gov/isecg
10+ international lunar exploration missions planned for next 10 years; many opportunities for demonstration of lunar GNSS capabilities.
NASA-USAF SSV Collaboration

- **Oct 13 2017: Joint NASA-USAF Memorandum of Understanding (MOU) signed**
  - MOU addresses civil Space Service Volume (SSV) requirements
  - Scope relevant to GPS IIIF acquisition process
    - Civil space early insight into Block IIIF design relevant to SSV performance
    - Access to Block IIF, III, and IIIF technical data

- **MOU results to-date:**
  - US civil space rep. from NASA supported GPS IIIF source selection team as SSV technical expert
  - Built positive, collaborative relationships with IIIF acquisition team; provided civil space insight continuing through design and production
  - NASA received released GPS IIF antenna pattern measurements per MOU and to support NASA Space Launch System need

  *MOU supports SSV signal continuity goals for future space users*
Automatic Flight Termination System (AFTS)

- Independent, self-contained subsystem mounted onboard a launch vehicle
- Flight termination / destruct decisions made autonomously via redundant Global Positioning System (GPS)/Inertial Measurement Unit (IMU) sensors
- Primary FTS for unmanned Range Safety Operations and being considered as Primary FTS for human space flight (Commercial Crew and SLS)
- Advantages:
  - Reduced cost—decreased need for ground-based assets
  - Global coverage (vehicle doesn’t have to be launched from a range)
  - Increased launch responsiveness
  - Boundary limits increase due to 3-5 second gain from not having Mission Flight Control Officer (MFCO)
  - Support multiple vehicles simultaneously (such as flyback boosters)

Enabling low cost, responsive, reliable access to space for all users
GAliileo Receiiver for the ISS (GARISS)

- **Objectives:**
  - Demonstrate combined GPS/Galileo (L5/E5a) navigation receiver on-orbit with upload of Software Radio waveform
  - Add waveform to Space Telecommunications Radio Systems (STRS) waveform repository

- **Approach/Benefits:**
  - Adapt existing Galileo PNT code to Software Defined Radio (SDR) inside ScAN Test Bed (STB) onboard International Space Station (ISS)
  - Demonstrate operations, conduct PNT experiments on ISS
  - Flexibility of SDR technology, STRS operating environment

- **Timeline:**
  - Initial discussions at International meetings (mid-2014)
  - Project formulation/export license (mid-2016)
  - Waveform design and development (late 2016-mid 2017)
  - Qualification and test the Galileo/GPS waveform (mid 2017-late 2017)
  - On-orbit testing and experiments (2018)
On-orbit waveform integration and testing

- Moved integration and testing to on-orbit operations April 2018
- Successful on-orbit acquisition, track and PVT solution
- Full function for GPS and Galileo processing established at qualification review (May 2018):
  - Acquisition and Time to First Fix (TTFF) requirements are met for Galileo and for combined GPS/Galileo
  - GPS-only on-orbit PVT availability > 20%
  - Galileo-only on-orbit PVT availability > 40%

- Achieved Combined Galileo/GPS PVT availability greater than 90%
- ~64m RMS positional error (3D)
- First-ever on-orbit direct acquisition of L5/E5a (no L1 aiding)
Real-Time Monitoring of GPS, GLONASS, BeiDou, Galileo, and QZSS by the GDGPS System

- Real-time (6 sec latency) orbit determination operations
- Additional hourly precise orbit determination operations
- Real time environmental monitoring: earthquakes, space weather, tsunamis
- Differential Code Bias Monitoring for all GNSS signals at high rate (especially needed to monitor GPS Flex Power)
Low cost, low power, high performance space GNSS receiver for POD and radio occultations (RO)
• Designed by JPL and built by Tyvak for initial use on GeoOptics’ CICERO RO Constellation
• Based on the PicoZed off the shelf OEM computer and JPL’s TriG GPS receiver design
• A NASA Class D version of this design has recently been completed

Key Cion Features:
• 3 antenna inputs with 4 down converters each
• 1.2 GHz Dual Core Arm processor
• 30cm X 10cm X 6cm
• 1 kg
• ~10 watts at 12 VDC
• Can be coupled with an external receiver to increase number of channels available for radio science

Successful operations on CICERO in orbit since March 2018, providing GPS and GLONASS radio occultation data
• Coupled with a Novatel GPS+GLONASS POD receiver
• Launch of additional CICERO satellites anticipated in 2018
Summary

- NASA is engaged in numerous space-based GNSS initiatives that are bearing great fruit for science and future mission development
- The NASA civil GNSS space user fleet is growing and expanding into new regimes
  - GRACE retired after 15 years of groundbreaking science
  - Space Launch System (SLS) has five upcoming cislunar exploration flights with GPS onboard
  - Restore-L and PACE will add to NASA’s extensive use of GPS in LEO
- NASA studies on cislunar use of GPS show high level of potential performance in lunar orbit with modest investments in receiver system capabilities.
  - The int’l Gateway project and the Global Exploration Roadmap provide numerous opportunities for in-depth study and inflight demonstrations.
- NASA’s GPS Antenna Characterization Experiment (ACE) has gathered and published the most extensive dataset on GPS antenna patterns for space users to-date, enabling a new series of high-fidelity mission simulations in the SSV.
- NASA technology developments have achieved numerous milestones:
  - Automatic Flight Termination System (AFTS) is providing operational GPS-based range safety since 2017
  - GARISS achieved the first direct acquisition of L5/E5a in space, and is exceeding its performance requirements in initial flight tests
  - JPL GDGPS system is providing real-time monitoring of GPS, GLONASS, Galileo, BDS, QZSS
  - New JPL Cion GNSS receiver for CubeSats is operational on-orbit
- We encourage GNSS providers to report on their activities to support the global space user community and enhance the utility of interoperable GNSS in space
GPS Signal Structure
Outline

I. GNSS signals
II. GNSS receivers
III. Measurements
IV. Time reference, orbits, coordinate frames
V. Navigation solution
GPS signal structure

• What is required of a radionavigation signal?
  1. Propagation delay between transmitter and receiver can be measured
  2. Transmitters can be distinguished, enabling geometric diversity
  3. Modulation allowing the signal to propagate through space

• For any signal $p(t)$ combined with Additive White Gaussian Noise (AWGN) $n(t)$,

$$p(t) + n(t)$$

correlation with a copy of $p(t)$ maximizes the output signal to noise ratio (SNR) (i.e., optimal estimator in the Maximum Likelihood sense), so $p(t)$ is designed to have a correlation shape that satisfies signal requirements 1 and 2

• Delay estimation

Consider a known, continuous-time signal $p(t)$ generated at the transmitter that arrives at the receiver with some delay $\tau$:

$$p(t - \tau)$$

In order to estimate $\tau$, a local replica of $p(t)$ is formed at the receiver with test delay $\tilde{\tau}$. The delay estimate, $\hat{\tau}$, is the test delay that maximizes the average (over $T_I$) of the inner product:

$$\hat{\tau} = \arg \max_{\tilde{\tau}} \frac{1}{T_I} \int_{t-T_I}^{t} p(\alpha - \tilde{\tau}) p(\alpha - \tau) d\alpha$$
GPS signal structure: code

- Autocorrelation in terms of alignment error, $\epsilon = \tau - \tilde{\tau}$:

$$R(\epsilon) = \frac{1}{T_I} \int_{t-T_I}^{t} p(\alpha - \tilde{\tau})p(\alpha - \tau) d\alpha$$

The ideal autocorrelation function would be

$$R(\epsilon) = \begin{cases} 
1 & \text{for } \epsilon = 0 \\
0 & \text{elsewhere} 
\end{cases}$$

- Multiple signals are required in order to form a position estimate, however. The trilateration problem relies on geometric diversity. One means of distinguishing transmitters is to minimize the cross correlation of signals from different transmitters:

$$R_x(\tau) = \frac{1}{T_I} \int_{t-T_I}^{t} p^i(\alpha)p^j(\alpha - \tau) d\alpha$$

$$R_x(\epsilon) = 0 \quad \forall \epsilon$$

- These auto- and cross-correlation properties could be achieved with infinitely long random sequences of $+1$ and $-1$, known at the transmitter and receiver, and unique to each transmitter.
GPS signal structure: code

- This is accomplished using **Pseudorandom Noise (PRN) codes**

  Must be deterministic and finite for practical implementation, but sufficiently long and noise-like to approximate the desired autocorrelation and cross-correlation properties

- GPS Coarse Acquisition Code (C/A code) solution: Gold codes (modulo-2 sum of two linear feedback shift registers)

  Periodic sequence of \{+1,-1\} pulses called chips, unique to each GPS satellite, length 1023 with period of 1 ms (i.e., \(f_{\text{chip}} = 1.023\ \text{MHz}\))
GPS signal structure: carrier

- Third navigation signal requirement: modulation allowing the signal to propagate through space

\[ p(t) \cos(2\pi f_{\text{carr}}t) \]

- Radio frequencies used for satellite navigation—must penetrate atmosphere

- Apparent frequency at the receiver is Doppler shifted due to the relative motion of the transmitter and receiver

\[ p(t - \tau(t)) \cos(2\pi f_{\text{carr}}(t - \tau(t))) \]

\[ p(t - \tau(t)) \cos(2\pi (f_{\text{carr}} + f_D)t - \theta(t_0)) \]

where \( \tau(t) = \dot{r}t + \tau(t_0) \) and \( f_D = -\dot{r}t = -\dot{r}(t)f_{\text{carr}}/c \)
GNSS carriers

- A variety of carrier frequencies are used by GNSS providers

Color code: **Blue**—open signals  **Red**—restricted or encrypted signals

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>L1: 1.57542 GHz</td>
</tr>
<tr>
<td>GLONASS</td>
<td>L1, L2, L5</td>
</tr>
<tr>
<td>SBAS</td>
<td></td>
</tr>
<tr>
<td>Galileo</td>
<td></td>
</tr>
<tr>
<td>QZSS</td>
<td></td>
</tr>
<tr>
<td>IRNSS</td>
<td></td>
</tr>
<tr>
<td>Proposed Beidou</td>
<td></td>
</tr>
<tr>
<td>Proposed GLONASS</td>
<td></td>
</tr>
</tbody>
</table>

- GPS L1 ($f_{L1} = 1.57542$ GHz) will be used as an example in this presentation
Finally, signal is also modulated with 12.5 minute navigation message, a 50 bps binary sequence containing time tags, GPS satellite ephemerides (i.e., transmitter locations), etc.

Time domain signal:

- Received L1 frequency signal from the $i$-th satellite

$$y^i(t) = \sqrt{2P_R} d^i(t - \tau^i(t)) p^i(t - \tau^i(t)) \cos(2\pi(f_{L1} + f_D^i)t + \theta^i(t_0)) + v^i(t)$$
GPS signal structure (continued)

• Received L1 signal from the \( i \)-th satellite

\[
y^i(t) = \sqrt{2P_R} d^i(t - \tau^i(t)) p^i(t - \tau^i(t)) \cos(2\pi(f_{L1} + f_D^i)t + \theta^i(t_0)) + v^i(t)
\]

• Frequency domain signal:

1. Data Message Spectrum

2. Data x PRN Code Spectrum

3. Data x Code x Carrier Spectrum

Composition of GPS signal in frequency domain [3]
GNSS receivers

- Receiver has three main tasks:
  
  1. **Acquisition:** Determine which satellites are visible and estimate the propagation delay and Doppler associated with each.
  
  2. **Tracking:** Refine the delay and Doppler estimates and track these features as they change over time.
  
  3. **Navigation:** Use measurements from all visible signals to estimate the receiver’s position and velocity.

- First the radio frequency signal is downconverted to an intermediate frequency (IF) for processing.
GNSS receivers: acquisition

- Acquisition seeks to determine whether a particular satellite is visible (via its unique PRN) and estimate its delay (modulo one code period, 1 ms) and Doppler.
- Correlation of an incoming signal with a local replica, mismatched in frequency and delay, forms what is known as an asymmetric ambiguity function:

\[
\chi(\tau, f_D) = \int_{t-T_I}^{t} p(\alpha)p^*(\alpha - \tau)e^{i2\pi f_D \Delta} d\alpha
\]

- Delay and Doppler values are tried over a search space. Correlation magnitudes are compared to the noise floor—if the carrier to noise spectral density exceeds a threshold, the signal is determined visible and the delay and Doppler at the correlation peak are used to seed tracking.
- Pre-detection integration time, \( T_I \), is an important parameter in detecting weak signals.
GNSS receivers: tracking

- Tracking seeks to refine the delay and Doppler estimates produced by acquisition
  1. Input signal is correlated with a local replica
  2. Correlation result is filtered to produce error terms that quantify the difference between the input and local signal
  3. A feedback process makes adjustments to the local signal replica according to the error terms
- In addition to converging on the input signal delay and Doppler parameters, the tracking of a dynamic signal allows for measurements of changing signal features and more accurate estimates of the signal to noise ratio
- Most receivers compute three correlations per signal: Early, Prompt, and Late
  - Phase of prompt corr. gives error signal for carrier tracking
  - Comparing size of Early and Late corr. gives error signal for PRN code tracking
- Coupled feedback loops DLL and PLL maintain lock on code and carrier signal parameters

GNSS measurements

- GNSS observables (i.e., receiver outputs)
  1. **Pseudorange**: propagation delay plus receiver clock bias (measured from the PRN code to a fraction of a chip: ~meter level accuracy)
  2. **Doppler**: measured frequency shift of the received carrier
  3. **Carrier phase**: measured fractional and accumulated whole cycle phase of the carrier (measured to small fraction of 19 cm cycle: ~mm precision)
  4. **C/N₀**: carrier to noise spectral density estimate in dB-Hz
GNSS measurements: pseudorange

- Pseudorange measured from the $i$-th satellite ("pseudo" because of receiver clock bias):

$$\rho^i = c(\tilde{t}_r - \tilde{t}_t^i)$$

- Time of reception according to receiver clock
- Time of transmission according to satellite clock
GNSS measurements: pseudorange

- Pseudorange measured from the $i$-th satellite ("pseudo" because of receiver clock bias):

\[
\rho^i = c(\tilde{t}_r - \tilde{t}_t^i)
\]

\[
= c((t_r + \delta t_r) - (t_t^i + \delta t_t^i)) + \epsilon
\]

- Transmission and receive times each expressed as a sum of the "true" time (i.e., the time according to a common time standard, such as GPST) plus an unknown bias.
GNSS measurements: pseudorange

- Pseudorange measured from the $i$-th satellite ("pseudo" because of receiver clock bias):

$$\rho^i = c(t_r - \tilde{t}_t^i)$$

$$= c((t_r + \delta t_r) - (t_t^i + \delta t_t^i)) + \epsilon$$

$$= c\delta t_r - c\delta t_t^i + |x - x_t^i| + Q^i + I_{L1}^i + T^i + \epsilon$$

- Propagation delay:

$$\tau^i = (r^i + Q^i + I_{L1}^i + T^i)/c$$

$r^i$ is the geometric range between the $i$-th transmitting satellite and the receiver, $|x - x_t^i|$

$Q^i$ is the satellite orbit error

$I_{L1}^i$ is the delay due to the ionosphere, a region of ionized gas in the upper atmosphere where the time varying density of free electrons and ions introduces a dispersive (frequency dependent) delay

$T^i$ is the delay due to the troposphere, the lowest region of the atmosphere, a non-dispersive medium consisting of dry gases and water vapor
GNSS measurements: pseudorange (cont.)

- Propagation delay:

\[ \tau^i = (r^i + Q^i + I_{L1}^i + T^i)/c \]

Acquisition and tracking measure code phase, i.e., ambiguous time of transmission modulo one code period

(1 ms for GPS C/A code, or approximately 300 km): \[ p(t - \tau) \]

The navigation message must be decoded to form a pseudorange

- Navigation message is organized into six second subframes, each beginning with an 8-bit Telemetry word (TLM) and Hand-over word (HOW), the latter of which contains the satellite time the subframe was transmitted

\[ \text{svtime_of_transmission} = \]
\[ \text{svtime_of_subframe} \]
\[ + \text{bits_since_subframe_start} \times 0.02 \]
\[ + \text{whole_cacodes_since_bit_start} \times 0.001 \]
\[ + \text{whole_chips_into_current_cacode} \times 1.023 \times 10^6 \]
\[ + \text{frac_code_phase_chips} \times 1.023 \times 10^6; \]

Calculation of pseudorange using four satellites [12]

Training on GNSS 2019 | Pathum Thani, Thailand
Navigation solution: typical GPS error budget

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Basic single frequency</th>
<th>Precise dual-freq, assisted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionosphere (&lt; 1000 km)</td>
<td>~3 m (single frequency, using broadcast model)</td>
<td>Dual frequency &lt;1 cm</td>
</tr>
<tr>
<td>Troposphere (&lt; 20 km)</td>
<td>0.1-1 m</td>
<td>1 cm level using estimators, advanced models</td>
</tr>
<tr>
<td>GPS orbits</td>
<td>&lt;2.0 m (broadcast ephem)</td>
<td>1 cm, Int. GNSS service (IGS)</td>
</tr>
<tr>
<td>GPS clocks</td>
<td>&lt;2.0 m (broadcast clock)</td>
<td>1 cm (IGS)</td>
</tr>
<tr>
<td>Multipath (“clean” environment)</td>
<td>0.5-1 m code</td>
<td>0.5-1 cm carrier</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>0.25-0.5 m code</td>
<td>1-2 mm carrier</td>
</tr>
<tr>
<td>RSS range error</td>
<td>4 m</td>
<td>2 cm</td>
</tr>
<tr>
<td>Typical GDOP</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>RSS solution error</td>
<td>8 m</td>
<td>4 cm</td>
</tr>
</tbody>
</table>

- Disclaimer: for illustration purposes only
GNSS measurements: pseudorange (cont.)

- **Propagation delay:**
  \[ \tau^i = (r^i + Q^i + I_{L1}^i + T^i)/c \]

- **Orbit error**
  Maintained to within ~1 m RSS by Control Segment

- **Ionosphere**
  Group delay for pseudorange due to the ionosphere:
  \[ I_{L1}^i = \frac{40.3}{f_{L1}^2} \text{TEC} \]

  where TEC is the Total Electron Content in a 1 m² column from the receiver the transmitter.

  Ionospheric delay can be corrected by using measurements from two frequencies (note frequency dependence, here we use GPS L1) or through a model that predicts TEC.

  GPS uses the Klobuchar model, in which four parameters (defined by eight numbers in the navigation message) are used to define the daily zenith variation at the ionospheric pierce point.
GNSS measurements: pseudorange (cont.)

- **Propagation delay:**
  \[ \tau^i = (r^i + Q^i + I_{L1}^i + T^i)/c \]

- **Troposphere**
  Not frequency dependent, wet (< 0.25 m, large variation) and dry (~2 m, small variation) components
  Corrected using models (e.g., Hopfield) that incorporate empirical corrections—typically average meteorological parameters for latitude, longitude, and season

- **Complete pseudorange expression:**
  \[ \rho^i = r^i + ct_{b,r} - ct_{b,s}^i + Q^i + I_{L1}^i + T^i + \epsilon^i \]

- **Relativity**
  Second-order Doppler shift: a clock moving in an inertial frame runs slower than a clock at rest
  Gravitational frequency shift: a clock at rest in a lower gravitational potential runs slower than a clock at rest in a higher gravitational potential
  GNSS space segment atomic clocks are offset to compensate for these effects—without correction satellite clocks would gain almost 40 microseconds per day (~10 km range error)
GNSS measurements: pseudorange (cont.)

- Propagation delay:
  \[ \rho^i = r^i + c t_{b,r}^i - c t_{b,s}^i + Q^i + I_{L1}^i + T^i + \epsilon^i \]

- Multipath

Reflected signals are received as delayed, attenuated replicas of the direct signal.

Correlation shape of the combined signal causes an error in the code tracking loop that depends on geometry, number and strength of reflections, and tracking loop design.

E.g., one signal, noncoherent DLL:

\[ \tau_M = \frac{\alpha^2 \delta + \alpha \delta \cos \psi}{1 + \alpha^2 + 2\alpha \cos \psi} \]
GNSS measurements (cont.)

- Measured **Doppler** shift is a combination of the changing geometric line of sight and the unknown receiver clock drift. Time derivative of the pseudorange:

  \[
  \dot{\rho}^i(t) = \dot{r}^i(t) + c \dot{t}^i_{b,r} - c \dot{t}^i_{b,s}
  \]

  Recall:
  \[
  f_D = \dot{r}(t) f_{carr}/c
  \]

- **Carrier phase**

  \[
  \phi^i(t) = -2\pi f_{carr} \rho^i(t)/c + 2\pi M
  \]

  Can be measured with much higher precision than code phase (i.e., pseudorange), ~cm for GPS, but ambiguous on the order of carrier cycles, 19 cm for GPS. Combine with code measurements or use for precise measurement of change (Accumulated Delta Range). Ionosphere also induces a delay, but opposite in sign relative to pseudorange, leading to a code/carrier divergence.

  Multipath also introduces an error in carrier phase measurements, as the geometry changes and the received reflections cycle through constructive and destructive interference with the direct signal. For a single reflection:

  \[
  \phi^i_M(t) = \left( 2\pi \Delta^i / \lambda_{carr} + \phi^i_R \right) \text{MOD} 2\pi
  \]
GNSS measurements (cont.)

- **Carrier phase (cont.)**

\[ \phi_M^i(t) = \left( \frac{2\pi \Delta^i}{\lambda_{carr}} + \phi_R^i \right) \text{MOD} 2\pi \]

Wall reflection

\[ \Delta = a + b = 2d \cos \theta \]

\[ \frac{\partial \phi_M^i}{\partial t} = - \left( \frac{4\pi d}{\lambda_{carr}} \sin \theta \right) \frac{\partial \theta}{\partial t} \]

Ground reflection

\[ \Delta = e - g = 2h \sin \theta \]

\[ \frac{\partial \phi_M^i}{\partial t} = \left( \frac{4\pi h}{\lambda_{carr}} \cos \theta \right) \frac{\partial \theta}{\partial t} \]

- **Carrier to noise spectral density** (C/N₀) is the signal power divided by the measurement noise power density. Unlike signal to noise ratio, SNR, this is independent of the receiver bandwidth B. It is a power to noise density per unit frequency, expressed in units dB-Hz:

\[ \frac{C}{N_0} = \text{SNR} + B = (P_R - B - N_0) + B \]
GNSS measurements: link budget

- GNSS link budget

The received signal power is a combination of power spatial density produced by the transmitter at the receiver and the effective area of the receive antenna:

\[ P_R = P_{T, rcvr} A_R \]

Effective area is a measure of an antenna’s ability to capture power in an electric field on the antenna from a certain direction, defined as:

\[ A_{eff} = G\lambda^2 / 4\pi \]

An isotropic antenna radiates power equally in all directions. At a given distance from the transmitter, \( R \), the power density is simply the transmitted power divided by the surface area of the sphere: \( P_T / (4\pi R^2) \). This accounts for spreading loss. Spreading loss can be offset by focusing the transmitted power in a particular direction, a property described by the transmit antenna gain, \( G_T \). The power density at the receiver is:

\[ P_{T, rcvr} = \frac{P_T G_T}{4\pi R^2} \]
GNSS measurements: link budget

- GNSS received power

  The power density at the receiver is:

  \[ P_{T,\text{rcvr}} = \frac{P_T G_T}{4\pi R^2} \]

  Thus for receive antenna gain \( G_R \) the received power is given by the Friis transmission formula:

  \[ P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi)^2} \]

- Noise power per frequency unit:

  \[ N_0 = kT_{Eff} \]

  Effective temperature used to characterize all noise, not just thermal

- Carrier to Noise Spectral Density

  \[ \frac{C}{N_0} = \frac{P_R}{N_0} = \frac{P_R}{P_N} \frac{1}{B} \]

  Typical values for GPS:

  \( T_E = 290 \text{ K} \rightarrow N_0 = -201 \text{ dBW-Hz}, \quad P_R = -156 \text{ dBW} \rightarrow C/N_0 = 45 \text{ dB-Hz} \)
Summary of GNSS receiver procedure

1. Point antenna toward sky signals
2. Receiver acquires and tracks 4 or more signals
3. At any given moment, latch code/carry phase observables for 4+ signals
4. Form pseudoranges from code/carry observables, correct with error models
5. Solve overdetermined system of nonlinear equations in least squares sense
6. Output position, velocity, and time (PVT) to user
Outline

I. GNSS signals

II. GNSS receivers

III. Measurements

IV. Time reference, orbits, coordinate frames

V. Navigation solution

\[ y^i(t) = \sqrt{2P_R} d^i(t - \tau^i(t)) p^i(t - \tau^i(t)) \cos(2\pi(f_{L1} + f_D^i)t + \theta^i(t_0)) + v^i(t) \]
GNSS time reference

- GNSS requires a common time scale for computing ranges
- GPS Time (GPST) is the operational time scale of GPS
- To keep satellites on GPST adequately, atomic clocks are required
  
  Corrections in the navigation message are used to synchronize satellites to GPST
  
  For example, to limit clock error to 1 m over 12 hrs requires drift $< 8 \times 10^{-14}$ s/s
- GPST coarsely steered to align with Universal Consolidated Time (UTC) as maintained by the US Naval Observatory via corrections in the navigation message
- **Traceability to UTC USNO enables precise time and frequency transfer on a global scale**

- Tidal friction and other processes that cause a significant redistribution of mass are slowing the Earth’s rotation, lengthening the solar day by $\sim 2$ ms / century
- UTC incorporates leap seconds to maintain alignment with sidereal time (UT1), but GPST does not. This difference is a persistent challenge for receiver designers and users.
GNSS orbits

- Satellites make great reference points
  Small number can provide global coverage
  They can be precisely located
- Orbital mechanics are well understood and satellite orbit determination is a refined science; for GPS, for example, the MCS estimates and predicts satellite orbits to less than 1 m
- Dual frequency observables from a network of monitor stations used to estimate orbits and satellite clock biases
- Each GPS satellite broadcasts its ephemeris (valid for 2-4 hours) and an almanac (subset of ephemeris parameters for every satellite in the constellation—not accurate enough for navigation, but accurate enough for a satellite search)
- International GNSS Service (IGS) and others maintain large networks of monitor stations, use advanced techniques to locate satellites with cm-level accuracy
GNSS Coordinate Frames

- Earth Centered Earth Fixed (rotating reference frame) versus Earth Centered Inertial (ECI)

Explanatory video: https://youtu.be/DbYapFLJsPA

ECEF and ECI reference frames animation [15]
GNSS Coordinate Frames

- GNSS orbit determination is performed in an inertial (non-rotating) frame
  
  Example: Earth Mean Equator and Mean Equinox of the J2000 epoch (January 1, 2000 at 12:00 TT), x-axis is aligned with the mean equinox, z-axis aligned with the Earth’s spin axis or celestial north Pole

- Terrestrial navigation is performed in an Earth-fixed frame (rotating with the Earth) for convenience to users
  
  Example: GPS uses the WGS84 frame, a 3-dimensional coordinate reference frame for establishing geodetic latitude, longitude, and heights for navigation. Defined by the US National Geospatial Intelligence Agency.
Navigation solution

- Position estimation with pseudorange

Want to estimate receiver position and clock bias at some instant in time:

\[ \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } b = c \delta t_{b,r} \]

Given \( N > 4 \) pseudorange measurements (corrected for transmitter clock bias):

\[ \rho^i = |\mathbf{x} - \mathbf{x}_t^i| + b + v^i \]

Standard approach is to solve as a non-linear least squares (NLLS) problem by Gauss-Newton method:

\[
\text{minimize} \quad J(\hat{\mathbf{x}}, \hat{b}) = \sum_{i=1}^{N} \left( \rho^i - \left( |\hat{\mathbf{x}} - \mathbf{x}_t^i| + \hat{b} \right) \right)^2
\]

1. Linearize about initial guess \((\hat{\mathbf{x}}_0, \hat{b}_0)\)
2. Solve linear least squares problem for \((d\hat{\mathbf{x}}, d\hat{b})\)
3. Set \( \hat{\mathbf{x}}_1 = \hat{\mathbf{x}}_0 + d\hat{\mathbf{x}}, \hat{b}_1 = \hat{b}_0 + d\hat{b} \)
4. Iterate
Navigation solution: DOP

- **In general**, when solving the linear least squares problem

\[ z = H\theta + v , \text{Cov}\{z\} = \sigma_z^2 I \]

- The covariance of the least squares solution \( \theta^* \) is

\[ \sigma_{\theta}^2 = \text{Cov}\{\theta^*\} = \sigma_z^2 (H^T H)^{-1} = \sigma_z^2 W \]

- \( W \) (the inverse Gramian matrix) transforms measurement noise into solution noise

- In GPS, the \( i \)-th row of \( H \) is

\[ h_i = [u_i^T, 1] \text{ with } u_i = \frac{x - x_i}{|x - x_i|} \text{ (unit vector from transmitter to receiver)} \]

- Thus, \( W \) is determined by the geometry of the visible transmitters. Dilution of Precision (DOP):

\[ \text{GDOP} := \sqrt{\sum_{i=1}^{4} W_{ii}}, \text{PDOP} := \sqrt{\sum_{i=1}^{3} W_{ii}}, \text{TDOP} := \sqrt{W_{44}} \]

- Examples

- If transmitters are in a plane, \( H \) is rank deficient and \( GDOP = \infty \)

- If transmitters are located at corners of a tetrahedron \( GDOP = \sqrt{3} \) (minimum for \( N = 4 \))
Improving performance

- Multi-frequency receivers
  - Eliminate ionosphere as an error source through “ionosphere-free combination”
- Carrier phase observables
  - Millimeter rather than meter level measurement noise and negligible multipath error
- Differential measurements: Receivers in close proximity can be used to cancel common error sources (e.g., Differential GPS, DGPS)
  - Ionosphere, troposphere, satellite orbit/clock can be cancelled by differencing measurements or solutions
  - Solution is relative
- Precision GNSS orbits and clocks
  - Available from global networks of reference receivers (e.g., International GNSS Service, ICG) for post-processing and in near real-time
- Augmentations
  - Additional transmitters and measurements enhance geometry
- Filtering
  - Incorporate dynamic constraints or additional measurement sources (e.g., inertial sensors)
- A combination of these techniques enables cm to mm level solutions
References

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Image references

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Image references (cont.)


Optimality of correlation with respect to SNR

An optimal decision statistic is one that maximizes the signal to noise ratio (SNR). This means that for an input signal \( x(t) = s(t) + n(t) \), with signal component \( s(t) \) and AWGN \( n(t) \), a linear time invariant (LTI) process, \( h(t) \), is needed that maximizes the SNR of the output \( z(t) = z_s(t) + z_n(t) \). In terms of the input and \( h(t) \), the output is

\[
z(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau + \int_{-\infty}^{\infty} h(\tau)n(t-\tau)\,d\tau,
\]

and the SNR of the output sampled at time \( T_I \) is

\[
SNR \equiv \frac{|z_s(T_I)|^2}{E[|z_n(T_I)|^2]},
\]

so the problem is to find

\[
h_M(t) = \arg\max_{h(t)} \frac{|z_s(T_I)|^2}{E[|z_n(T_I)|^2]}.
\]

By Parseval’s theorem,

\[
|z_s(T_I)|^2 = \left| \int_{-\infty}^{\infty} h(\tau)s(T_I-\tau)\,d\tau \right|^2 = \left| \int_{-\infty}^{\infty} H(f)S(f)e^{i2\pi T_If}\,df \right|^2
\]

and

\[
E[|z_n(T_I)|^2] = R_{z_n,\bar{z}_n}(0) = \int_{-\infty}^{\infty} S_{z_n,z_n}(f)\,df = \int_{-\infty}^{\infty} |H(f)|^2 S_{nn}(f)\,df.
\]
Optimality of correlation (cont.)

For white noise, \( S(f) = N_0 / 2 \), so

\[
SNR = \frac{\left| \int_{-\infty}^{\infty} H(f)S(f)e^{i2\pi f T_I} df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}.
\]

By the Schwartz inequality, for any two square-integrable functions \( f \) and \( g \)

\[
\left| \int f(t)g(t) dt \right|^2 \leq \int |f(t)|^2 dt \cdot \int |g(t)|^2 dt,
\]

with equality if and only if \( f(t) = \lambda g^*(t) \), where \( \lambda \) is a scalar and the asterisk indicates complex conjugate. Signal to noise ratio is therefore bounded:

\[
SNR \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df} = \frac{2}{N_0} \int_{-\infty}^{\infty} |S(f)|^2 df.
\]

Equality holds when \( H(f) = \lambda S^*(f)e^{-2\pi f T_I} \). This corresponds to

\[
h(t) = \mathcal{F}^{-1} \{ H(f) \} = \lambda s(T_I - t).
\]

Thus, SNR is maximized when \( h(t) \) is a scaled, flipped, time-delayed copy of the input signal.

- See Introduction to Digital Communications by Michael B. Pursley