Assessment of Combined Integrity Algorithms

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OUTLINE

- Overview
- User Equations
- Comparison of Integrity Measures
- Combined Algorithm
- Simulation Results
- Conclusions

Definition of integrity

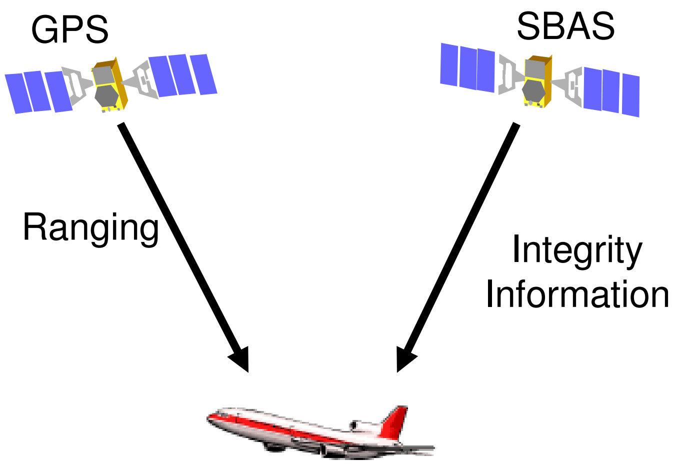
Integrity denotes the measure of trust placed in the correctness of the information provided by navigation systems.

Users may determine their integrity by

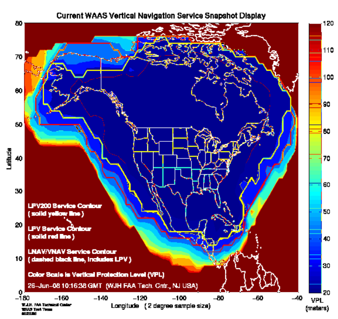
- Receiver autonomous algorithms (RAIM)
- External integrity data sources (e.g. SBAS)
- Integrity data provided within the navigation data message (e.g. Galileo)

Overview

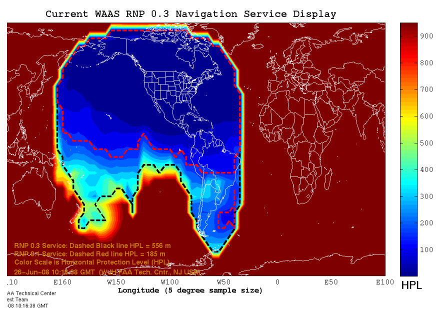
GPS + SBAS



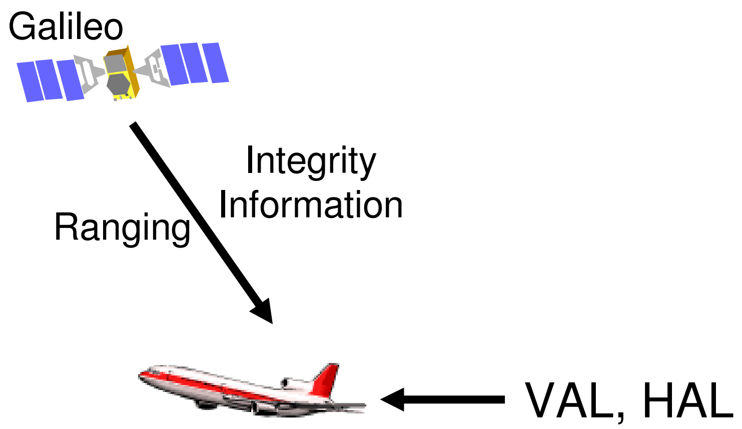
VPL



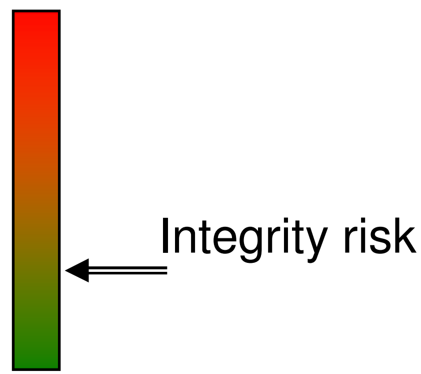
HPL



Galileo



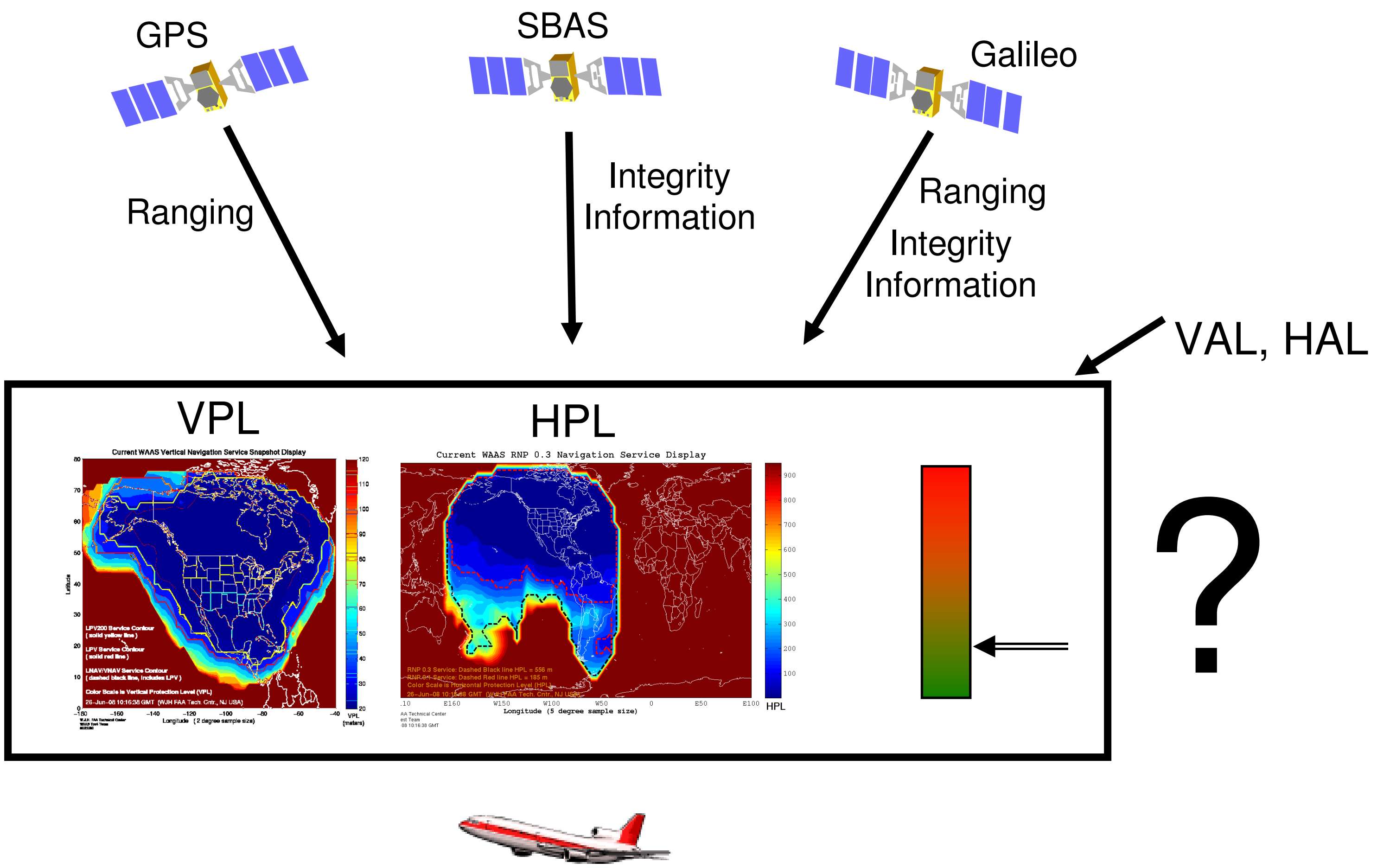
Systemlayout



User information

Overview

GPS + SBAS + Galileo



User Equations

GPS + SBAS

Input quantities on user side

•Geometry between GPS satellites and user derived from observations of the GPS satellites

•User differential range error σ_{UDRE}, transmitted by SBAS satellite

•Grid ionospheric vertical error σ_{GIVE}, transmitted by the SBAS satellite

•Tropospheric error σ_{tropo} derived from the model defined within the Radio Technical Commission For Aeronautics (RTCA) publication, *Minimum Operational Performance Standards (MOPS) For Global Positioning System/Wide Area Augmentation System Airborne Equipment*, RTCA DO-229D

•Error of airborne receiver errors σ_{air}, calculated depending on receiver properties and models defined within RTCA DO-229D

User Equations

GPS + SBAS

Algorithm:

- Compute measurement variances

$$\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2$$

- Transform variances to the position domain
  - Law of error propagation
  - Topocentric geometry matrix
  - Weight matrix

$$\mathbf{G}_i = [-\cos El_i \sin Az_i \quad -\cos El_i \cos Az_i \quad -\sin El_i \quad 1]$$

$$\begin{bmatrix} d_{east}^2 & d_{EN} & d_{EU} & d_{ET} \\ d_{EN} & d_{north}^2 & d_{NU} & d_{NT} \\ d_{EU} & d_{NU} & d_U^2 & d_{UT} \\ d_{ET} & d_{NT} & d_{UT} & d_T^2 \end{bmatrix} = \left(\mathbf{G}^T \mathbf{W} \mathbf{G}\right)^{-1}$$

- Compute semi-major axis of horizontal error ellipse

$$d_{major} = \sqrt{\frac{d_{east}^2 + d_{north}^2}{2} + \sqrt{\left(\frac{d_{east}^2 - d_{north}^2}{2}\right)^2 + d_{EN}^2}}$$

- Give HPL and VPL as multiples of the computed variances

$$VPL = K_{V,PA} \cdot d_U$$

$$HPL = \begin{cases} K_{H,NPA} \cdot d_{major} & \textit{en route} \text{ inclusive non-precision approach} \\ K_{H,PA} \cdot d_{major} & \text{precision approach mode} \end{cases}$$

User Equations

Galileo

Input quantities on user side

•Geometry between Galileo satellites and user position derived from observations of the Galileo satellites

•SISA as prediction of the expected SIS error, transmitted by the Galileo satellites

•SISMA comprising the accuracy of the monitoring process of the SIS error at the Galileo ground segment, transmitted by the Galileo satellites

•Integrity flag transmitted by the Galileo satellites

•Horizontal alarm limit (HAL) and vertical alarm limit (VAL), chosen by the user according to the designated application (e.g. landing approach)

•Remaining errors

User Equations

Galileo

Algorithm:

•Calculate overall Integrity risk P_{HMI} as sum consisting of fault-free and faulty mode allocation tree, split into four independent calculated parts

$$P_{HMI}(VAL,HAL) = P_{IR,V} + P_{IR,H}$$

$$P_{HMI}(VAL,HAL) = P_{IR,V,FaultFreeMode} + P_{IR,V,FaultyMode} + P_{IR,H,FaultFreeMode} + P_{IR,H,FaultyMode}$$

$$P_{HMI}(VAL,HAL) = 1 - erf\left(\frac{VAL}{\sqrt{2}\cdot\sigma_{u,V,FF}}\right) + e^{-\frac{HAL^2}{2\xi_{FF}{}^2}} + \frac{1}{2}\cdot\sum_{j=1}^{N} P_{fail,sat_j}\cdot\left(\left(1-erf\left(\frac{VAL+\mu_{u,V}}{\sqrt{2}\cdot\sigma_{u,V,FM}}\right)\right)+\left(1-erf\left(\frac{VAL-\mu_{u,V}}{\sqrt{2}\cdot\sigma_{u,V,FM}}\right)\right)\right) + \sum_{j=1}^{N}\left(P_{fail,sat_j}\cdot\left(1-\chi^2_{2,\delta_{u,H}}cdf\left(\frac{HAL^2}{\xi_{FM}{}^2}\right)\right)\right)$$

•Compute satellite to user geometry

$$M_{topo} = N_{topo}{}^T\cdot\left(H^T P H\right)^{-1} H^T P$$

•Derive needed variances, e.g.

$$\sigma_{u,V,FF}{}^2 = \sum_{i=1}^{N} M_{topo}[3,i]^2\cdot\left(SISA_i^2 + \sigma_{u,L,i}{}^2\right)$$

User Equations

Conclusions(I)

- SBAS + GPS integrity concept defines that all GPS satellites considered healthy by the SBAS ground segment are working nominally and may be used by the user
- Both integrity concepts use vertical and horizontal components to assess the measure of integrity
- Analogy: $\underbrace{\left(SISA_i^2 + \sigma_{u,L,i}{}^2\right)}_{Galileo} \hat{=} \underbrace{\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2}_{GPS+SBAS}$
- Fault free allocation tree within the Galileo integrity concept implicitly equals the SBAS + GPS integrity concept except for the allocated confidence intervals and the representation of the final result
- Final assessment of user integrity yields one major difference between the two concepts. SBAS + GPS concept uses HPL and VPL (in meters) derived from fixed error allocations, Galileo uses the probability P_{HMI} with confidence intervals chosen by the user in terms of HAL and VAL (in meters).

Conclusions(II)

- With the transition of the SBAS + GPS integrity algorithm definition contained within RTCA DO-229C to the newer version D, the rational for the definition of the K values was changed.
  - Correction in Overbounding argumentation carried out
  - See paper *“Does the HPL Bound The HPE”,* Christian Tiberius and Dennis Odijk, Navitec 08
  - Corresponding argumentation used in baseline Galileo concept up to now
- As a consequence, only SBAS + GPS HPL and VPL are now conservative estimates, while the conservatism in the range domain is no longer guaranteed.
- Protection levels and integrity risks at the alert limit are mathematically an inversion of the same context but cannot be compared directly due to the different allocations -> solution strategies needed

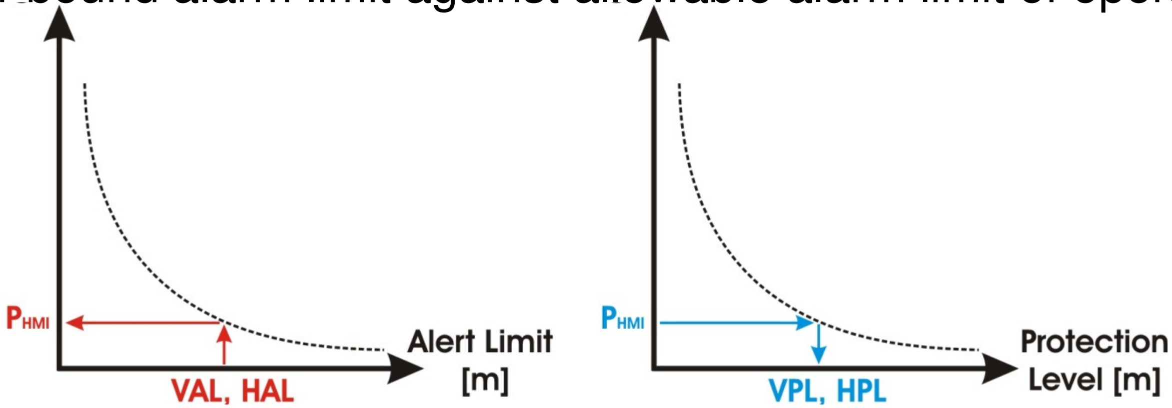
Direct and indirect Integrity Formulation

Direct problem (Galileo case):

- Specify alarm limit of operation
- Compute associate integrity risk
- Compare computed integrity against allowable integrity risk

Inverse Problem (SBAS + GPS case)

- Specify on system level allowable integrity risk for the user equation part of allocation tree
- Compute upper bound for alarm limits not resulting in integrity risks violating the specified allowable risk
- Compare upper bound alarm limit against allowable alarm limit of operation



Compare Integrity Measures

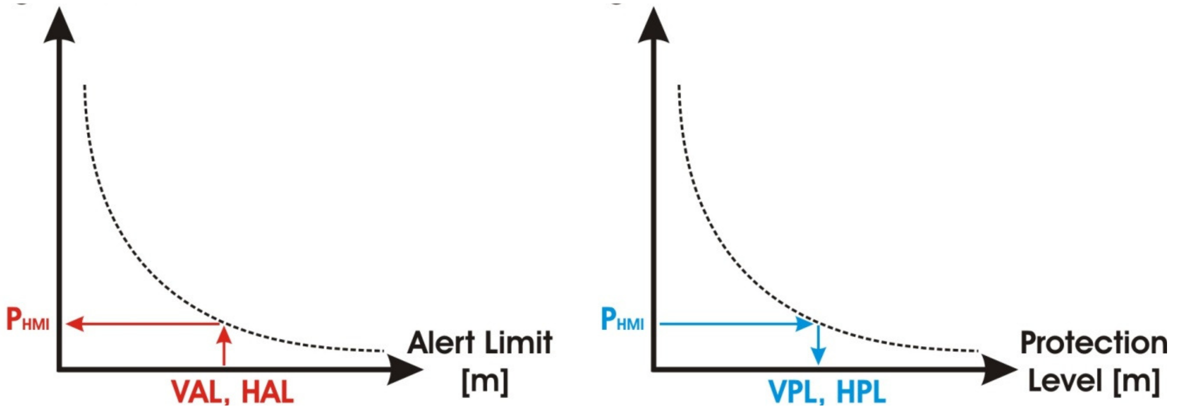
Solving strategies (I)

- Integrity risk functions are separated into independent horizontal and vertical components

$$\tilde{P}_{HMI} = P_{HMI,H}(HAL) + P_{HMI,V}(VAL)$$
$$= P_{HMI}(HAL,\infty) + P_{HMI}(\infty, VAL)$$

- Resulting in solvable set of optimization problems

$$HPL = \max_{HAL \in R_+} \arg P_{HMI,H}(HAL) \leq IR_H$$
$$VPL = \max_{VAL \in R_+} \arg P_{HMI,V}(VAL) \leq IR_V$$
$$IR_H + IR_V \leq IR$$



Solving strategies (I)

Resolve HPL first

- Check, if $P_{HMI,H}(HAL) \leq IR$
- Allocate integrity risks to $IR_V = IR - IR_H = IR - P_{HMI,H}(HAL)$
- Resolve $VPL = \max_{VAL \in R} \arg P_{HMI,V}(VAL) \leq IR_V, \; HPL = HAL$

Resolve VPL first

- Check, if $P_{HMI,V}(VAL) \leq IR$
- Allocate integrity risks to $IR_H = IR - IR_V = IR - P_{HMI,V}(VAL)$
- Resolve $HPL = \max_{HAL \in R} \arg P_{HMI,H}(HAL) \leq IR_H, \; VPL = VAL$

Solving strategies (I)

Fixed Allocation

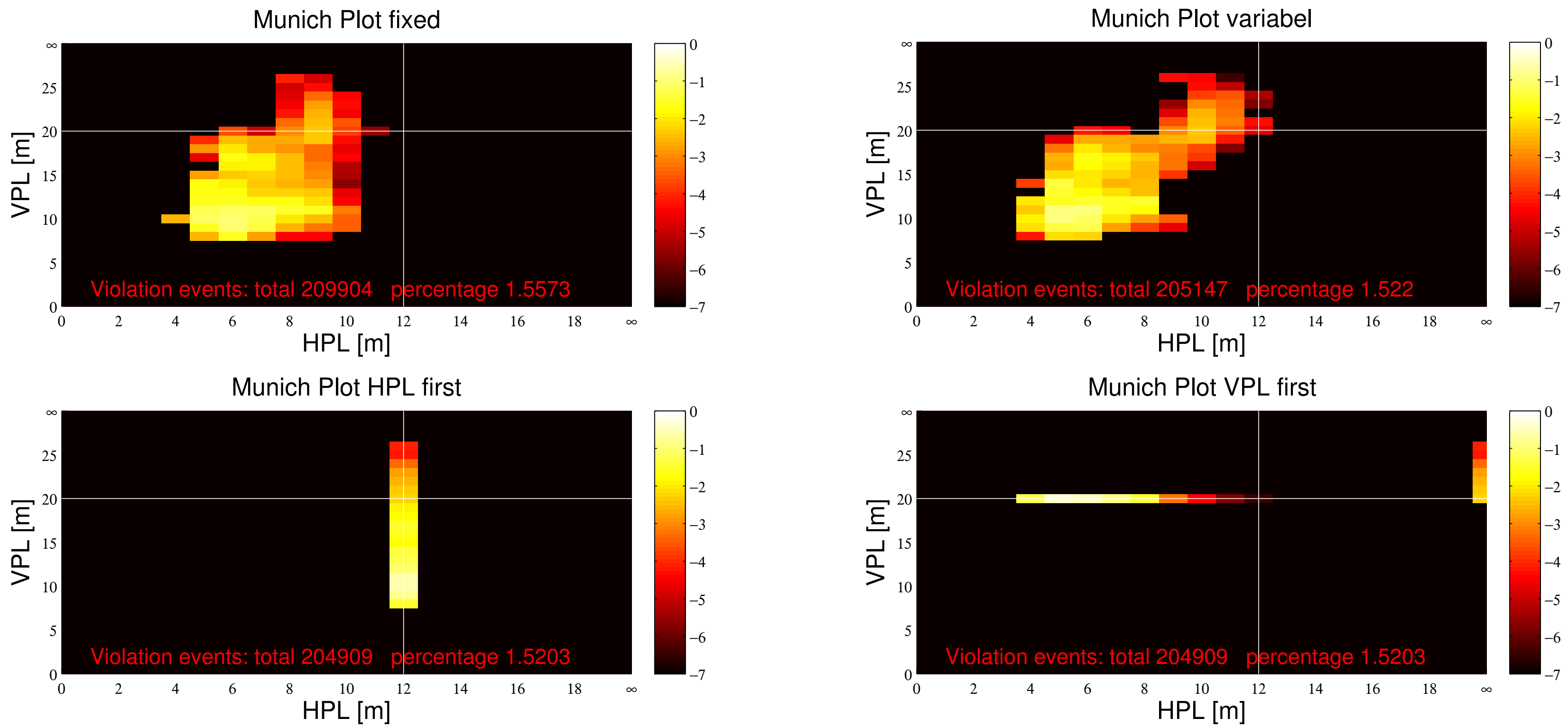
- Split IR fixed to IR_H and IR_V
- Resolve $HPL = \max_{HAL \in R} P_{HMI,H}(HAL) \leq IR_H$ and $VPL = \max_{VAL \in R} P_{HMI,V}(VAL) \leq IR_V$

Geometry dependent variable Allocation

- Split IR proportional to associated integrity risks IR_H and R_V at the alert limits
- Resolve $IR_H = \frac{P_{HMI,H}(HAL)}{P_{HMI,H}(HAL) + P_{HMI,V}(VAL)} \cdot IR$ and $IR_V = \frac{P_{HMI,V}(VAL)}{P_{HMI,H}(HAL) + P_{HMI,V}(VAL)} \cdot IR$

No analytical solution for solving strategies -> use of root finding algorithm

Solving strategies (I)



Munich plots for different solving strategies (I)

Compare Integrity Measures

Solving strategies (II)

SBAS Integrity Risk Formulation – en route computation

- No vertical guidance
- Horizontal protection level described as a quantile of the Raleigh distribution with respect to d_{major}

$$P_{HMI,G,H}(HAL) = \chi_2^2 cdf\left(\frac{HAL^2}{d_u^2}\right)$$

SBAS Integrity Risk Formulation – precision approach

- Vertical protection level described as a quantile of the Normal distribution with respect to d_U

$$P_{HMI,G,V}(VAL) = 2 \cdot \int_{VAL}^{\infty} \frac{1}{d_u\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x}{d_u}\right)^2\right) dx$$

- Horizontal guidance described as a quantile of the Normal distribution with respect to d_{major}

$$P_{HMI,G,H}(HAL) = 2 \cdot \int_{HAL}^{\infty} \frac{1}{d_{major}\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x}{d_{major}}\right)^2\right) dx$$

Combined Algorithm

Possibilities on user side to think of

Using Galileo SISA within SBAS + GPS integrity concept, neglect SISMA	**No** *SBAS assumes on user level all satellites indicated healthy to be healthy, in Galileo integrity concept one satellite may be faulty*
Independent parallel calculation and a posteriori integration	**Possible but suboptimal solution** *Integration of two independent results means averaging -> worse outcome compared to "true" combined algorithm*
Using data provided by SBAS within Galileo integrity concept	**Possible** *According to RTCA DO229 SBAS assumes all satellites indicated healthy to be healthy -> per definition SISMA is 0. Results in an additional geometry independent integrity risk contribution.*

Combined Algorithm

Procedure

- Computation of measurement variances and biases following the description of each system
- Single Point Positioning for combined Measurements
  - 4 Parameter estimation (inter system bias known)
  - 5 Parameter estimation (inter system bias estimated)
- Application of the law of error propagation deriving variances and noncentralities on the position domain
- Integrate the tails of the probability density functions starting from respective alarm limits
- Sum up all integrity risk components including the unallocated error of SBAS user equations

Algorithm equals Galileo user equation with P_{fail}=0 for GPS satellites with an additional fixed risk component

Combined Algorithm

Currently weak point

- Since transition of RTCA Do 229 from issue C to issue D the choice of the K-factors is “somewhat arbitrary”
- Conservatism only guarantied in position domain

Possible solutions

- Free inside view into SBAS Ground segment algorithms
- Generation of conservative estimations in range domain, e.g. slightly degradation factor
- Additional Data provided by SBAS satellites (L2 frequency incorporating new integrity data?)

Combined Algorithm

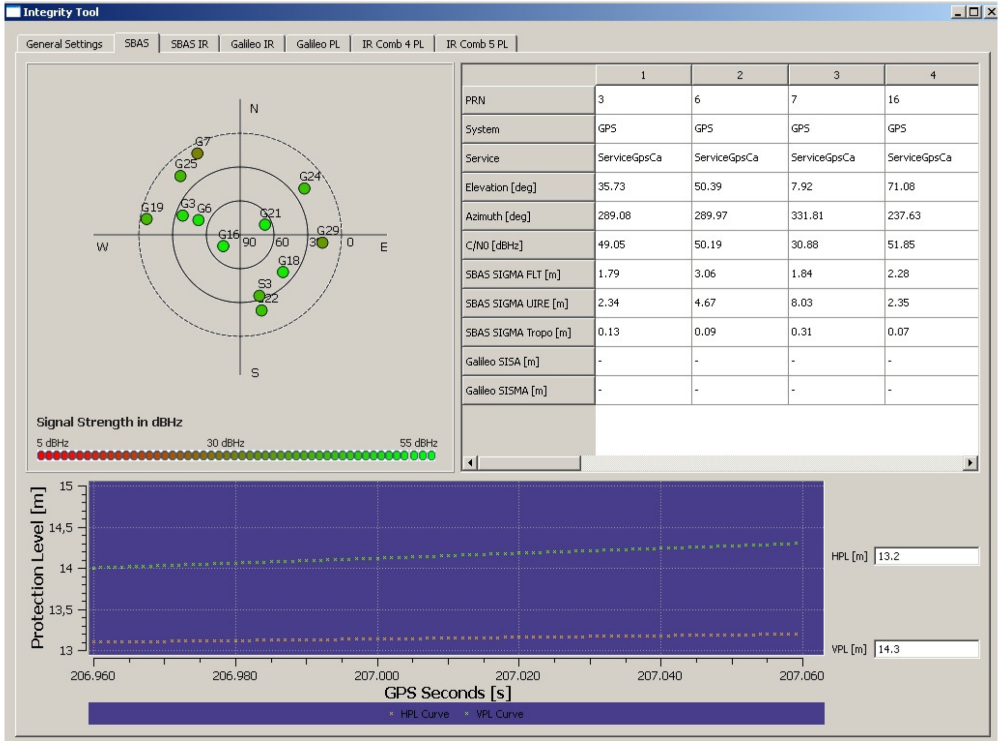
Integrity Simulation Tool

Key Functionality

- SBAS data processing conforming DO-229
- SBAS performance estimation on a global scope
- Galileo integrity performance estimation
- Combined algorithm performance estimation

Additional Functionality

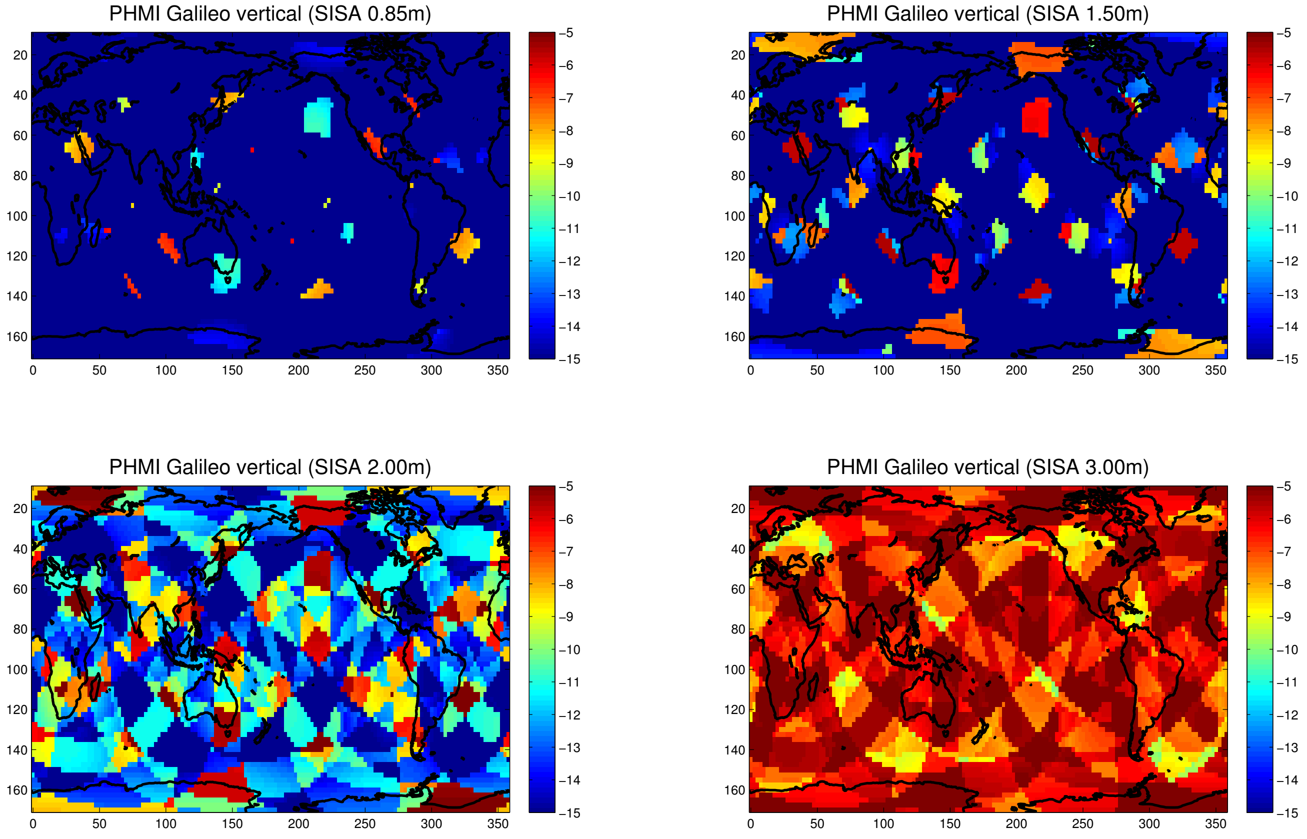
- Raw measurement generation
- Random measurement degradation
- Flexible data interfaces
  - Ground Segment to Space Segment
  - Ground Segment to User Segment



Integrity Tool works on real time data generated by the Integrity Simulation Tool

Simulation Results

Single epoch



Receiver noise Galileo	0.00 m
Tropospheric noise	0.05 m
Orbit and Clock noise Galileo (SISA)	variable
Galileo ionosphere factor	0.02
SISMA	0.8 m
HAL	12 m
VAL	20 m

Comparison of Galileo only vertical integrity risks for different SISA values

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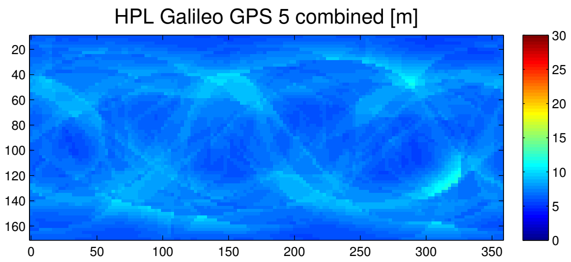
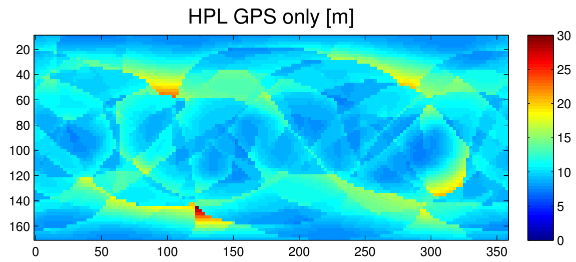
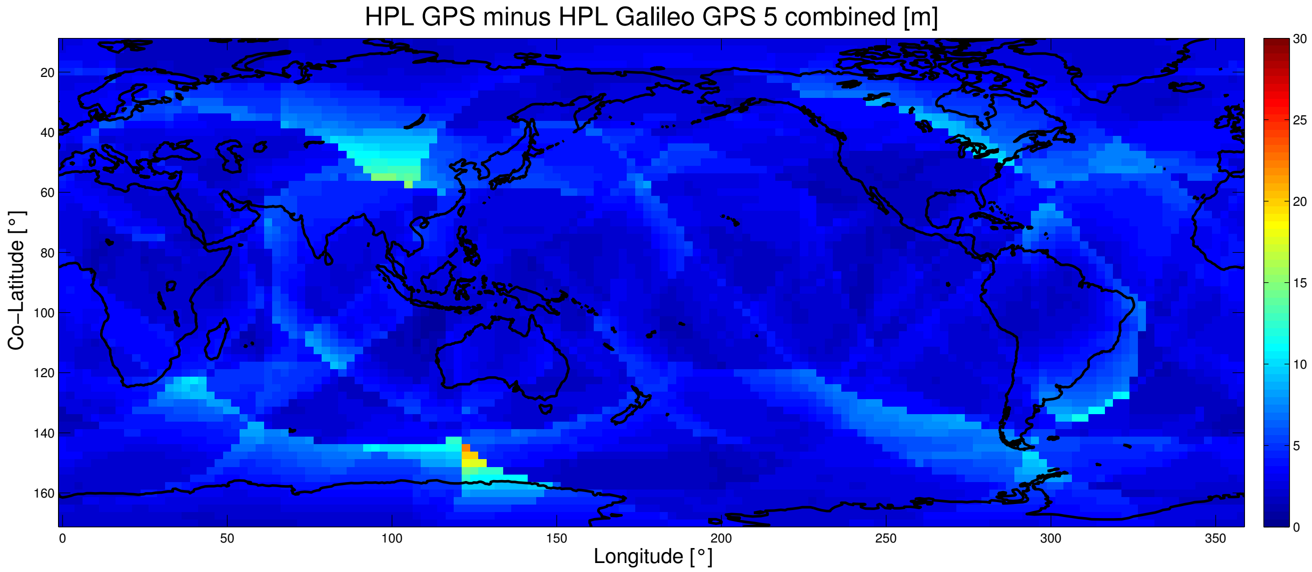
Simulation Results

Single epoch







Receiver noise (GPS and Galileo)	1.00 m
Tropospheric noise	0.05 m
Orbit and Clock noise GPS (σ_{UDRE})	2.00 m
Orbit and Clock noise Galileo (SISA)	2.00 m
GPS ionosphere factor	0.02
Galileo ionosphere factor	0.02
SISMA	0.8 m
HAL	12 m
VAL	20 m

Comparison between combined algorithm and SBAS + GPS, assuming equal magnitude of measurement errors

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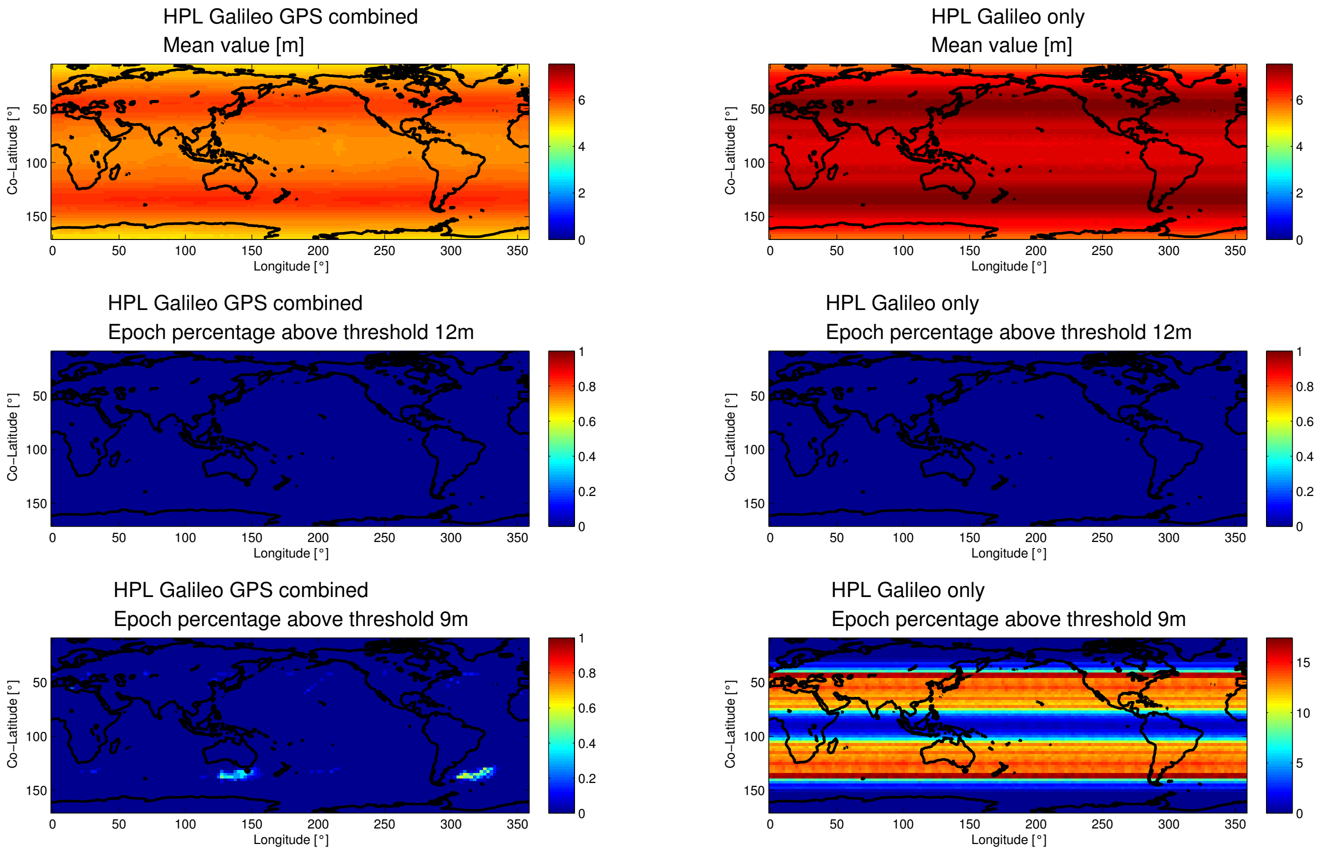
Simulation Results

Timeline analysis



Receiver noise (GPS and Galileo)	1.00 m
Tropospheric noise	0.05 m
Orbit and Clock noise GPS (σ_{UDRE})	2.00 m
Orbit and Clock noise Galileo (SISA)	2.00 m
GPS ionosphere factor	0.02
Galileo ionosphere factor	0.02
SISMA	0.8 m
HAL	12 m
VAL	20 m

Comparison of combined algorithm and Galileo-only algorithm for different thresholds

Conclusions

- Planned performance of the Galileo system is challenging and highly dependent on the clock and orbit accuracy
- Inverting strategies for shifting protection level formulations to integrity risk formulations provide a better comparability of Galileo integrity with SBAS + GPS integrity
- The conservative joint of the different integrity risk allocation trees results in an additional additive and geometry-independent integrity risk component for all GPS satellites. The simulation results demonstrate that this additive term in the combined algorithm does not deplete the geometry and redundancy induced advantages. Consequently, combined use of integrity information outperforms either single system used alone

It is the solely decision of all involved service providers to jointly define and certify combined integrity processing schemes, combined equipment regulatory and combined procedures.

Thanks for your attention

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The presented work was done within the framework of the UniTaS IV project founded by the Bundesministerium für Wirtschaft und Technologie (BMWi) administered by the Agency of Aeronautics of the DLR in Bonn (FKZ 50 NA 0734).