Optimising the Minimal Detectable Bias in GNSS Positioning Fault Detection

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Credibility and Ubiquitous Positioning

- Ubiquitous Positioning, Indoor Navigation and Location Based Service

  \- *Need To Be Credible* -

- But what does this mean?
  - Positioning perspective
    - Precision , DOPs, Confidence Regions…
    - Reliability/Integrity, MDBs, PLs, External Reliability...
  - Maps perspective
    - Accuracy, Precision, Reliability….
    - Currency…

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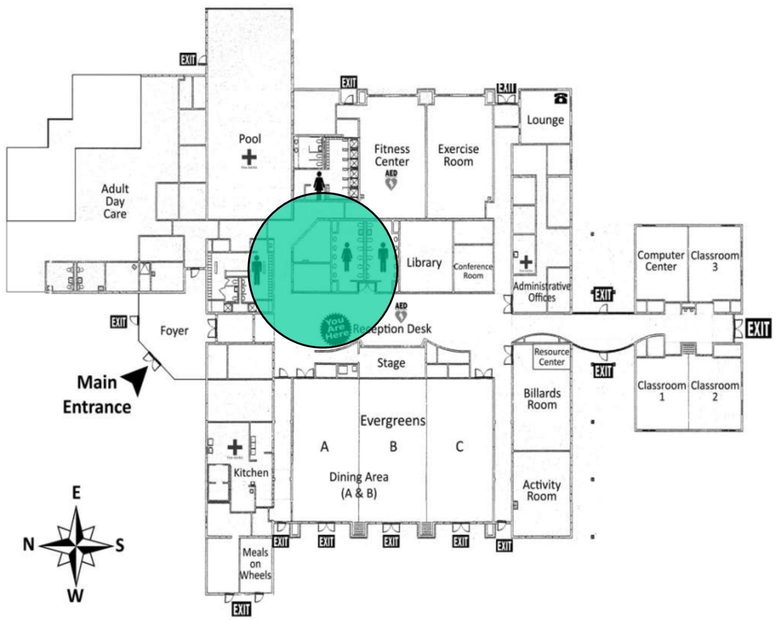
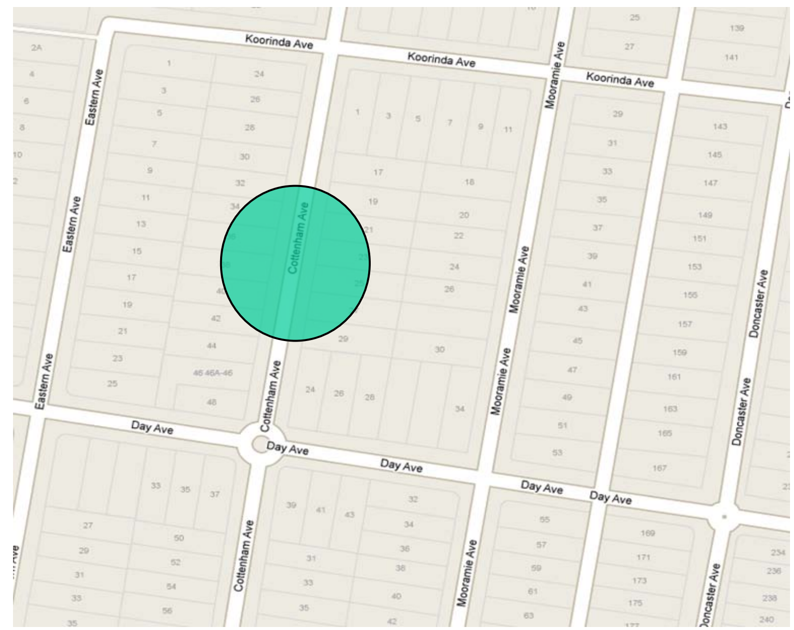
Credibility From Positioning Perspective

- Want meaningful information without being misleading





- But also desire ubiquitous positioning

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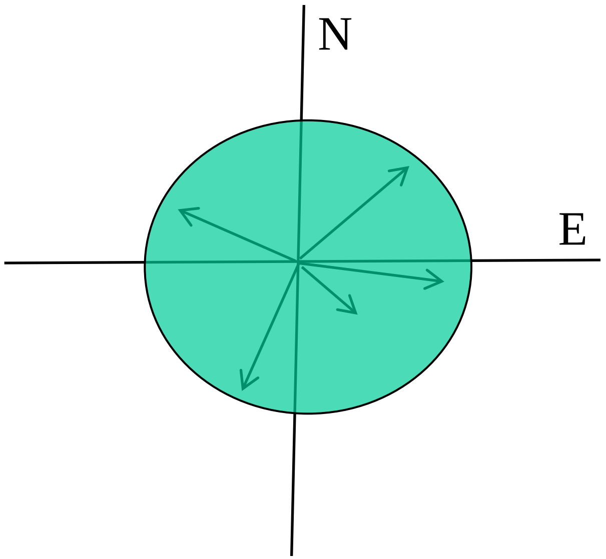
Credibility From Geodesy Perspective

- Creditability provided in Reliability
- Procedure
  - Design measurements, geometry to achieve Internal and External Reliability requirements
  - Take measurements
  - Least Squares and outlier testing
  - Remeasure
- Procedure driven to provide reliability at lowest cost



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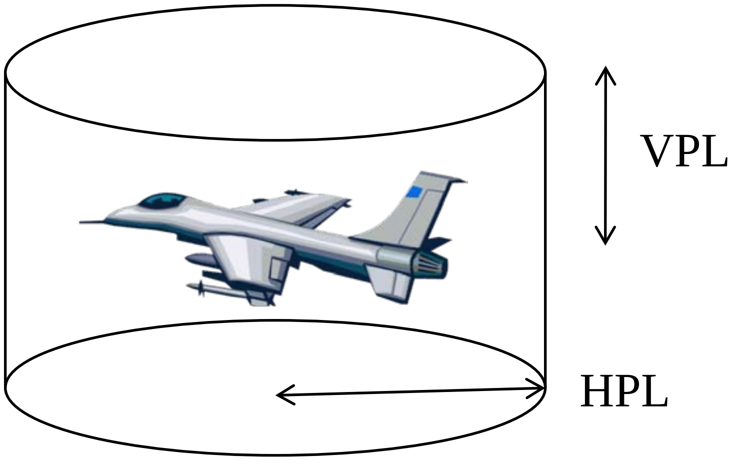
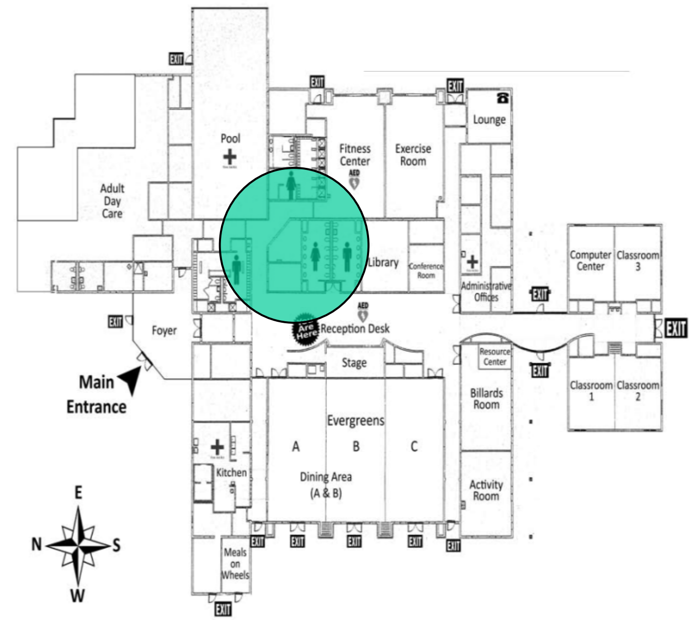
Credibility From Aviation Perspective

- Creditability provided in Integrity/RAIM
- Procedure
  - Use geometry as is
  - If unsatisfactory use other positioning technologies
- Driven by many requirements to be satisfied
  - Integrity
  - Continuity
  - Availability



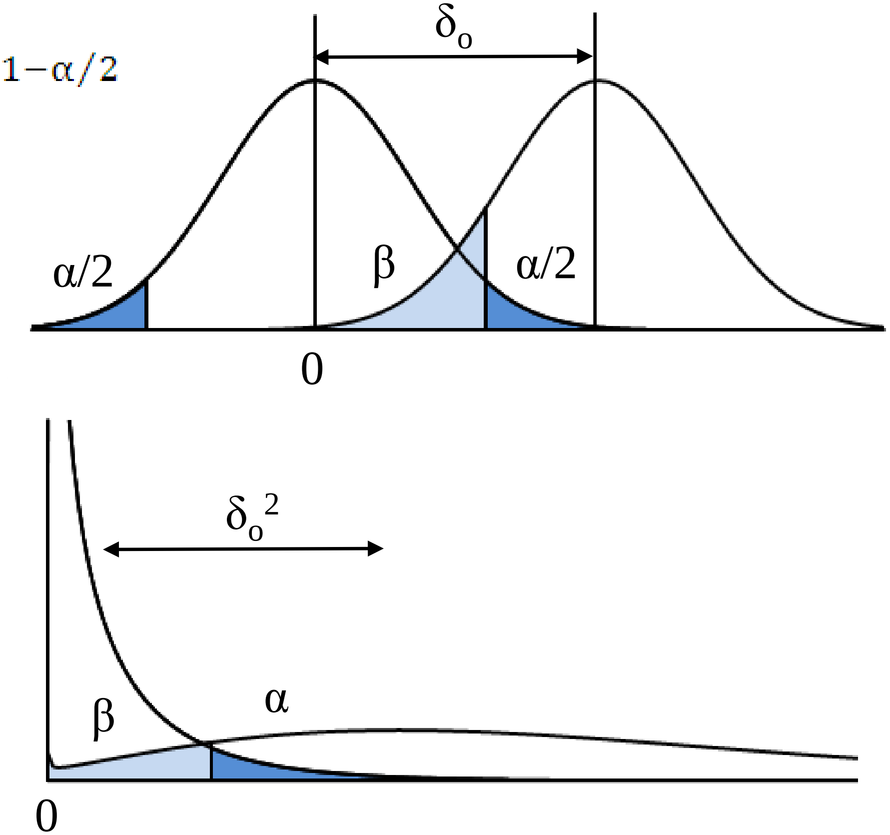
Credibility From Ubiquitous Perspective

- Credibility from a more general ubiquitous perspective
- Procedure
  - Use geometry as is
  - If unsatisfactory may relax requirements
- Driven to provide a position that always has the required integrity



The Conventional Outlier Test Method

- Set α first, $\alpha = 1 - \sqrt[n]{1 - \mathrm{P_{FA}}}$
- $w_i = \dfrac{\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{Q}_v \boldsymbol{P} \boldsymbol{\ell}}{\sigma_0 \sqrt{\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{Q}_v \boldsymbol{P} \boldsymbol{h}_i}} \sim \mathrm{N}(0,1)_{1-\alpha/2}$
- $\beta = \mathrm{P_{MD}}$
- $\delta_o \approx \mathrm{N}(0,1)_{1-\alpha/2} - \mathrm{N}(0,1)_{\beta}$
- $\chi^2_{1-\alpha,1} = \chi^2_{\beta,1,\delta_o{}^2}$



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The Conventional Outlier Test Method

- $$\delta = \mathrm{E}\left\{\frac{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{\ell}}{\sigma_0\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{h}_i}}\right\} = \frac{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{h}_i}\nabla s_i}{\sigma_0}$$
- $$\mathrm{MDB}_i = \nabla_{\mathrm{o}} s_i = \frac{\delta_{\mathrm{o}}\sigma_0}{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{h}_i}}$$
- $$\mathrm{PL}_i = \frac{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{h}_i}}{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{h}_i}}\sigma_0\delta_{\mathrm{o}}$$
- Final PL is the maximum PL_i
- Position has integrity when PL<AL

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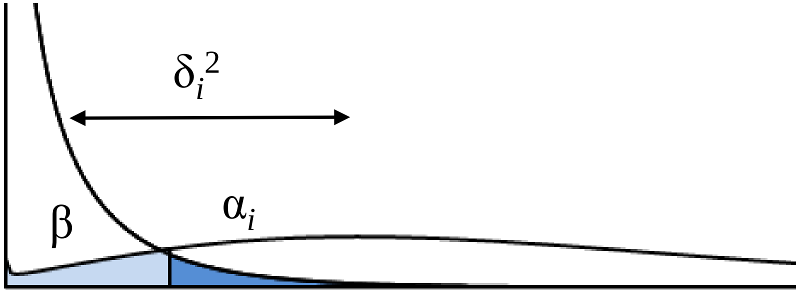
The Optimised Outlier Testing Method

- $$\mathrm{PL}_i = \frac{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{PA}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Ph}_i}}{\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{PQ}_v\boldsymbol{Ph}_i}}\sigma_0\delta_o$$
- **Set $\mathrm{PL}_i = \mathrm{AL}$**
- $$\delta_i = \frac{\mathrm{AL}\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{PQ}_v\boldsymbol{Ph}_i}}{\sigma_0\sqrt{\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{PA}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Ph}_i}}$$
- $\beta = \mathrm{P}_{\mathrm{MD}}$
- $\chi^2_{1-\alpha_i,1} = \chi^2_{\beta,1,\delta_i^2}$



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The Optimised Outlier Testing Method

- Use horizontal and vertical α_i in outlier test

$$w_i = \frac{\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{Q}_v \boldsymbol{P} \boldsymbol{\ell}}{\sigma_0 \sqrt{\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{Q}_v \boldsymbol{P} \boldsymbol{h}_i}} \sim \mathrm{N}(0,1)_{1-\alpha_i/2}$$

- If horizontal tests pass then horizontal integrity
- If vertical tests pass then vertical integrity
- Continuity probability can also be estimated via P_{FA} as

$$\mathrm{P}_{\mathrm{FA}} \leq 1 - \prod_{i=1}^{n} (1 - \alpha_i)$$

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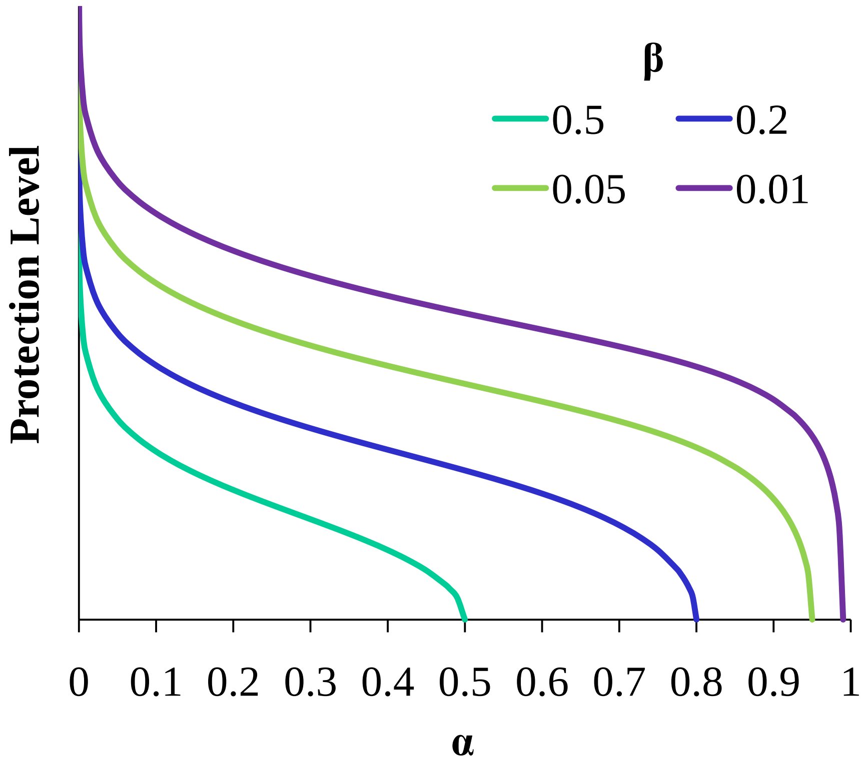
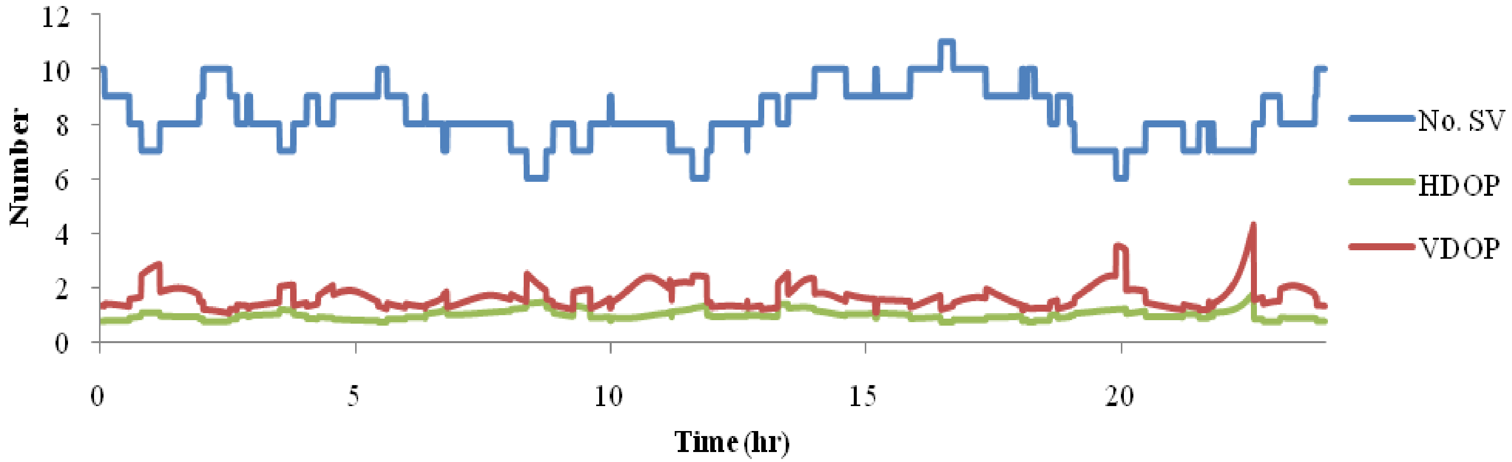
Reductions in PL as α Increases

- Based on a Single Bias
- $0<PL_i<\infty$
- $0<\delta_i<\infty$
- $0<\alpha_i<1-\beta$

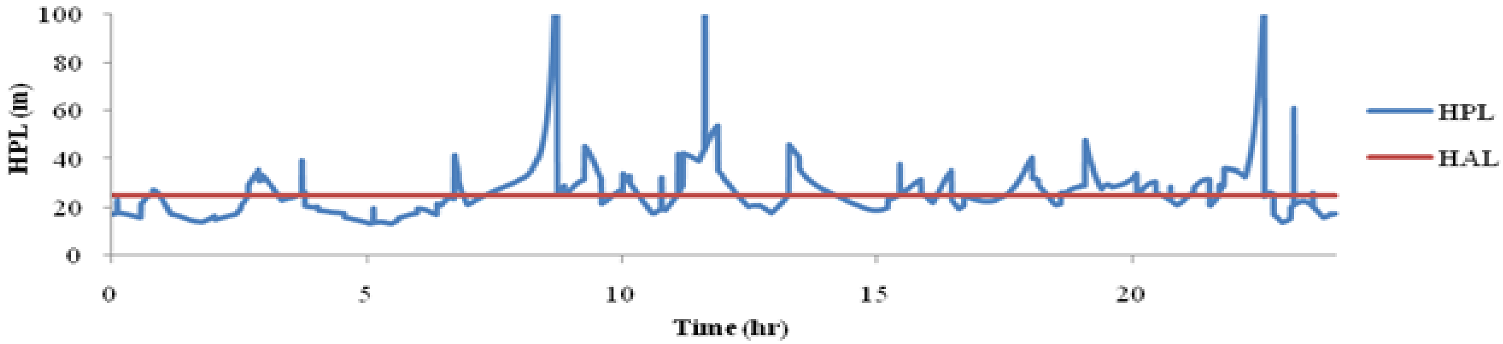


Example with 24hrs of GPS Data

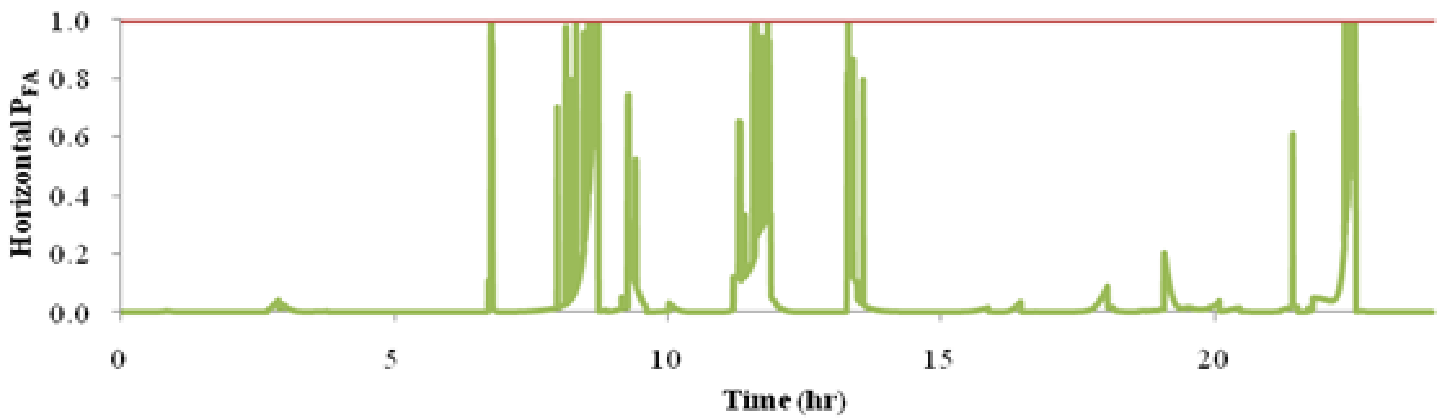


- **Set HAL=25m and VAL=50m**
- Set β=0.2
- In conventional FDE set P_{FA}=0.01

Horizontally Based on a Single Bias

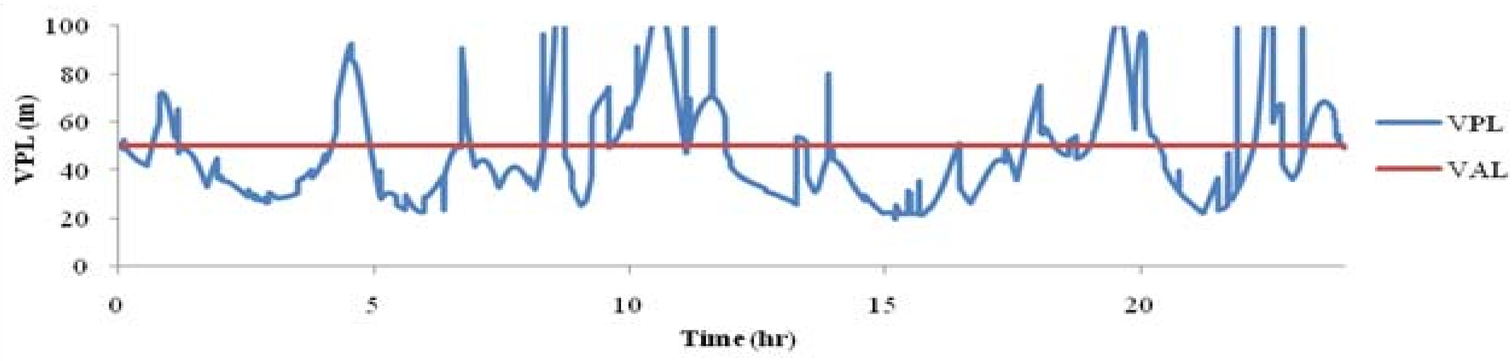


- A position with integrity only 55% of the time

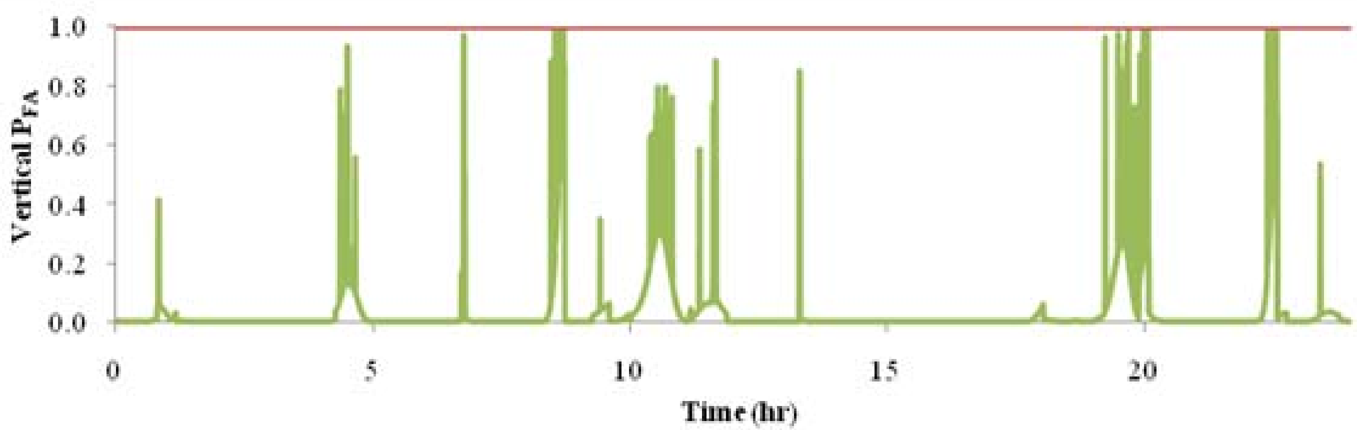


- A position with integrity 99% of the time

Vertically Based on a Single Bias



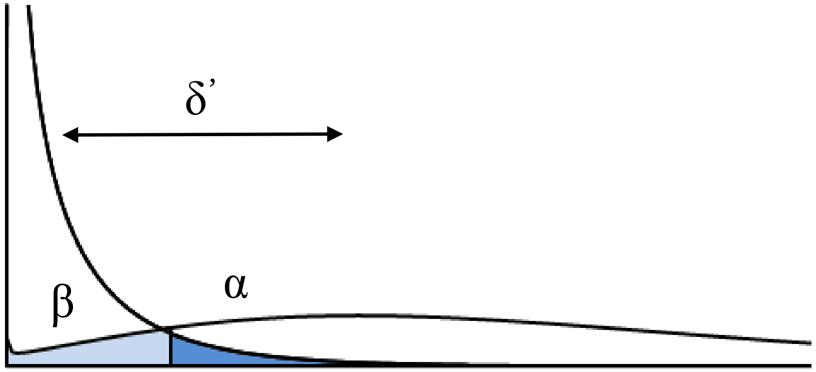
- A position with integrity only 65% of the time



- A position with integrity 99% of the time

The Conventional Outlier Test Method

- Set α first, $\alpha = \mathrm{P}_{\mathrm{FA}}\left(\frac{2(n-2)!}{n!}\right)$
- $w^2 = \frac{\boldsymbol{\ell}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{H}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{\ell}}{\sigma_0{}^2} \sim \chi^2_{1-\alpha,\,2}$
- $\beta = \mathrm{P}_{\mathrm{MD}}$
- $\chi^2_{1-\alpha,2} = \chi^2_{\beta,2,\delta'}$
- $\delta' = \frac{\boldsymbol{\nabla S}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{H}\boldsymbol{\nabla S}}{\sigma_0{}^2}$
- $\delta'_{\mathrm{o}} = \frac{\boldsymbol{\nabla}_{\mathrm{o}}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{H}\boldsymbol{\nabla}_{\mathrm{o}}\boldsymbol{S}}{\sigma_0{}^2}$



No unique MDB

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The Conventional Outlier Test Method

- $\mathrm{PL} = \sqrt{\boldsymbol{\nabla}_{\mathbf{o}}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{H}\boldsymbol{\nabla}_{\mathbf{o}}\boldsymbol{S}}$
- No unique PL - Desire maximum PL
- $\mathrm{PL}_{\mathrm{Max}} = \sigma_0\sqrt{\delta'_{\mathrm{o}}\lambda_{\mathrm{Max}}}$

$$(\boldsymbol{U}^{\mathrm{T}})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{H}\boldsymbol{U}^{-1}\boldsymbol{u} = \lambda\boldsymbol{u}$$

$$\boldsymbol{U}^{\mathrm{T}}\boldsymbol{U} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{Q}_v\boldsymbol{P}\boldsymbol{H}$$

- Final PL is the maximum $\mathrm{PL}_{\mathrm{Max}}$
- Position has integrity when PL<AL

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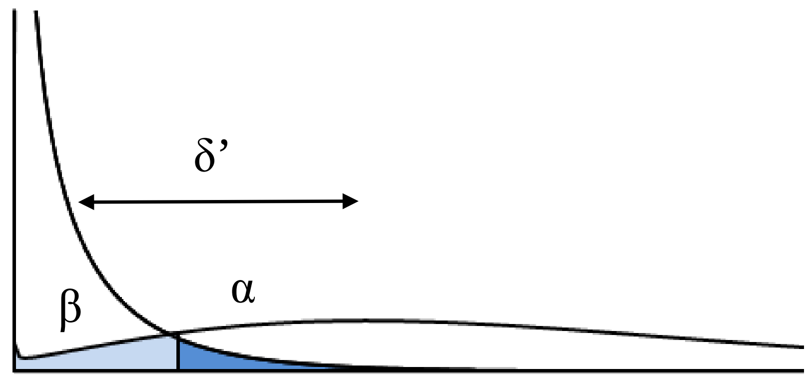
The Optimised Outlier Testing Method

- $PL_{Max} = \sigma_0 \sqrt{\delta'_o \lambda_{Max}}$
- **Set PL_{Max} = AL**
- $\delta' = \dfrac{(AL)^2}{{\sigma_0}^2 \lambda_{Max}}$
- $\beta = P_{MD}$
- $\chi^2_{1-\alpha,2} = \chi^2_{\beta,2,\delta'}$
- $w^2 = \dfrac{\boldsymbol{\ell}^T \boldsymbol{P Q_v P H} (\boldsymbol{H}^T \boldsymbol{P Q_v P H})^{-1} \boldsymbol{H}^T \boldsymbol{P Q_v P} \boldsymbol{\ell}}{{\sigma_0}^2} \sim \chi^2_{1-\alpha, 2}$
- If horizontal tests pass then horizontal integrity
- If vertical tests pass then vertical integrity



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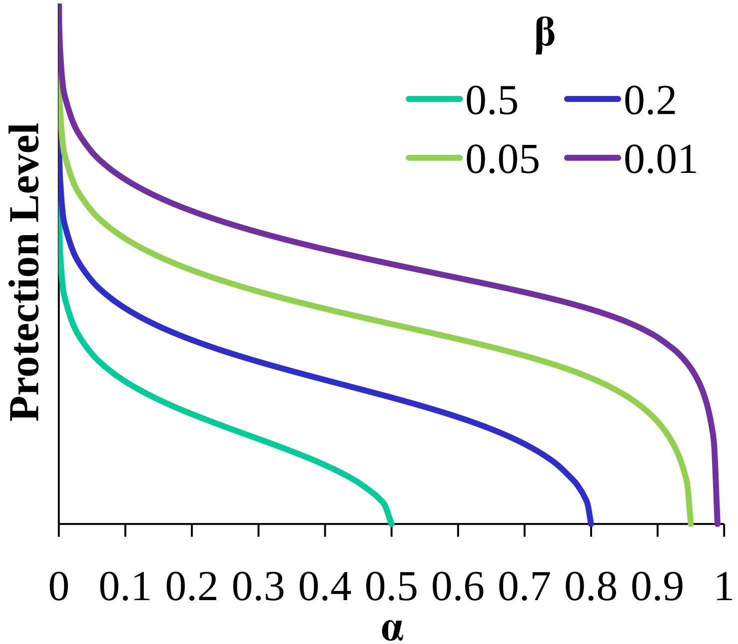
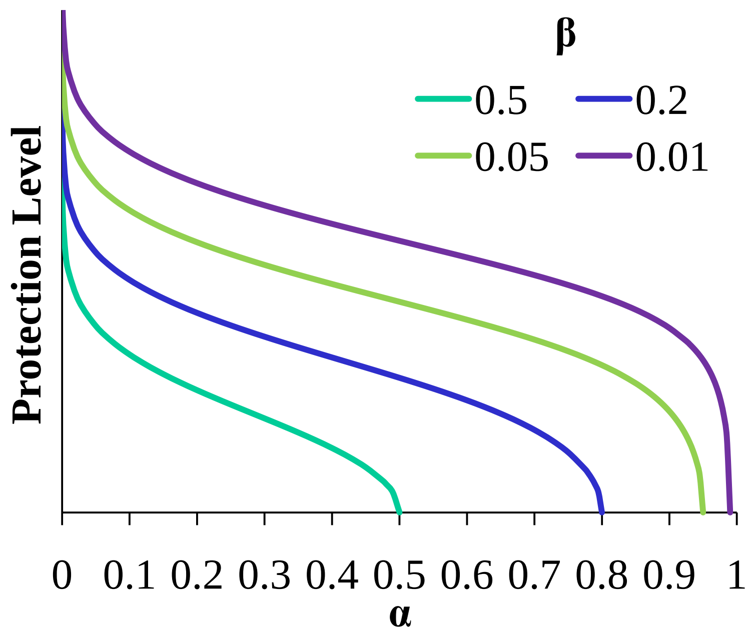
Reductions in PL as α Increases

Single Bias



Two Biases



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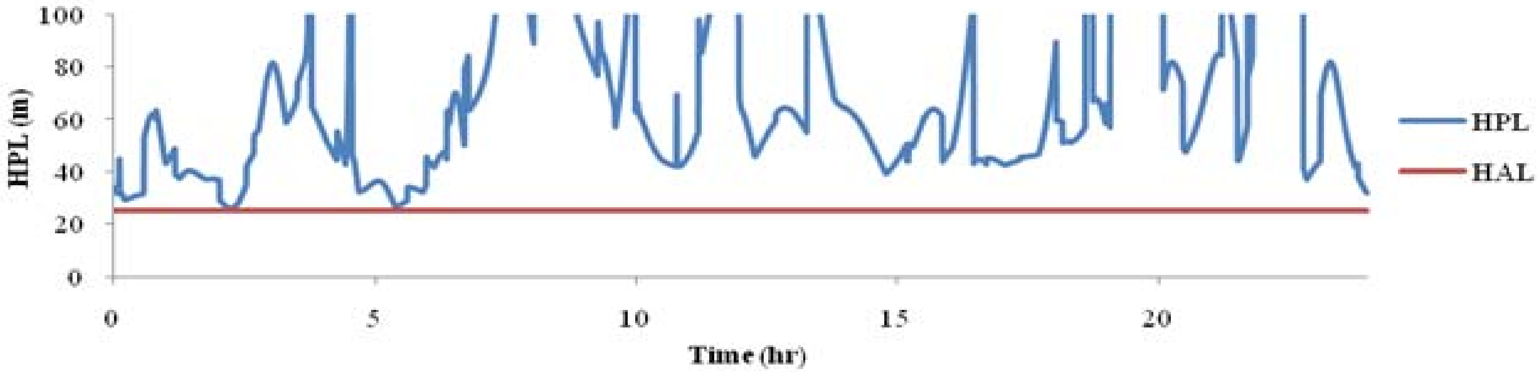
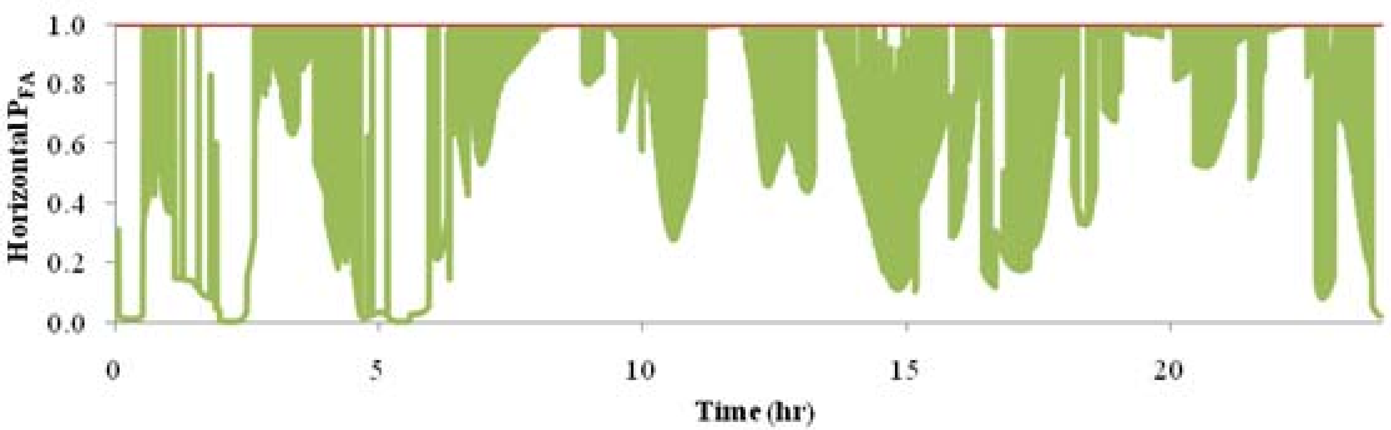
Horizontally Based on Two Biases

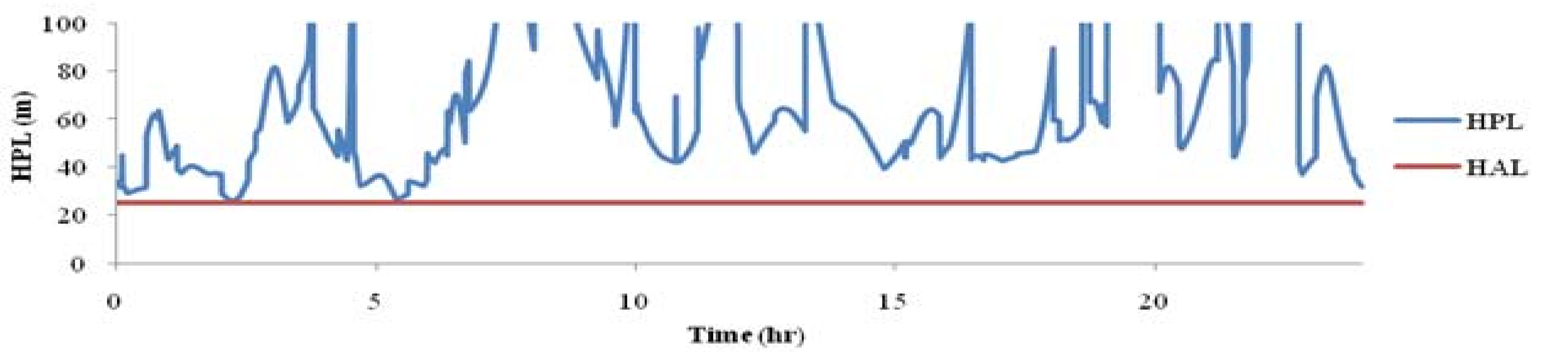


- No position with integrity

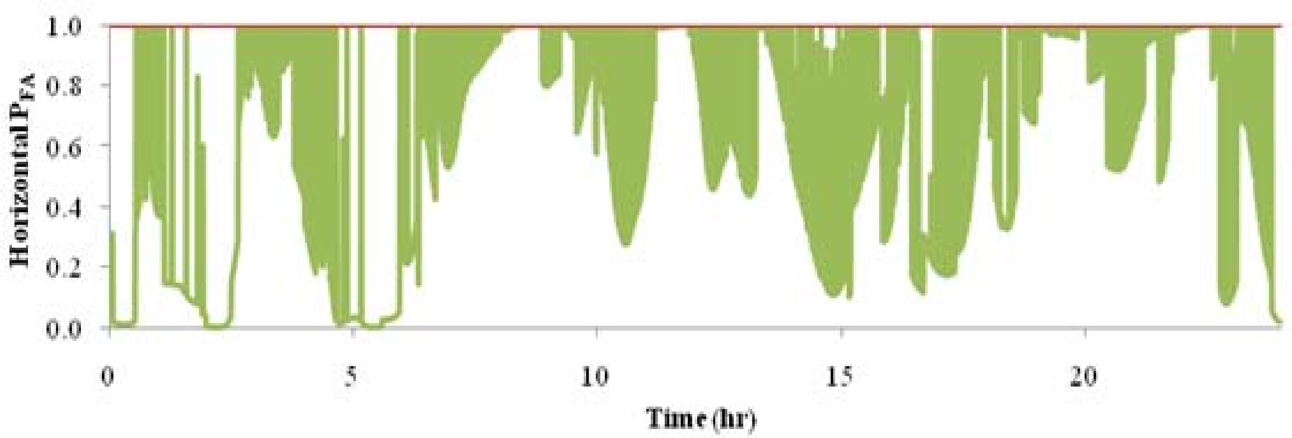


- A position with integrity 87% of the time

Vertically Based on Two Biases



- A position with integrity only 2% of the time



- A position with integrity 88% of the time

Conclusion

- Ubiquitous positioning desires to have a position that always has a defined integrity
- The way to achieve this is via setting PL=AL first then determine α
- PLs can be significantly reduced by changing α
- The results have showed a significant increase in the percentage of time that a position with a given integrity can be provided, particularly for two biases

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Thank you for your attention!