Global Terrestrial Reference Systems and Frames

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OUTLINE

• What is a Terrestrial Reference System (TRS), why is it needed and how is it realized?
• Concept and Definition
• TRS Realization by a Frame (TRF)
• International Terrestrial Reference System (ITRS) and its realization: the International Terrestrial Reference Frame (ITRF)
• ITRF2008 Geodetic & Geophysical Results
• How to access the ITRF?
• GNSS associated reference systems and their relationship to ITRF:
  – World Geodetic System (WGS84)
  – Galileo Terrestrial Reference Frame (GTRF)
Defining a Reference System & Frame:

Three main conceptual levels:

- **Ideal Terrestrial Reference System (TRS):**
  Ideal, mathematical, theoretical system

- **Terrestrial Reference Frame (TRF):**
  Numerical realization of the TRS to which users have access

- **Coordinate System:** cartesian (X,Y,Z), geographic (λ, φ, h), ...

  - The TRF is a materialization of the TRS inheriting the mathematical properties of the TRS
  - As the TRS, the TRF has an origin, scale & orientation
  - TRF is constructed using space geodesy observations
Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)
Defined in an Euclidian affine space of dimension 3:

Affine Frame (O,E) where:

O: point in space (Origin)
E: vector base: orthogonal with the same length:
  - vectors co-linear to the base (Orientation)
  - unit of length (Scale)

\[
\lambda = \| \vec{E}_i \| \quad \text{for } i = 1, 2, 3
\]

\[
\vec{E}_i \cdot \vec{E}_j = \lambda^2 \delta_{ij}
\]

\[
(\delta_{ij} = 1, \quad i = j)
\]
Terrestrial Reference Frame in the context of space geodesy

• **Origin:**
  – Center of mass of the Earth System

• **Scale (unit of length):** SI unit

• **Orientation:**
  – Equatorial (Z axis is approximately the direction of the Earth pole)
Transformation between TRS (1/2)

\[
X_2 = T + \lambda \mathbb{R} X_1
\]

7-parameter similarity:

Translation Vector \( T = (T_x, T_y, T_z)^T \)

Scale Factor \( \lambda \)

Rotation Matrix \( \mathbb{R} = R_x. R_y. R_z \)

\[
R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos R_1 & \sin R_1 \\ 0 & -\sin R_1 & \cos R_1 \end{pmatrix}
\]

\[
R_y = \begin{pmatrix} \cos R_2 & 0 & -\sin R_2 \\ 0 & 1 & 0 \\ \sin R_2 & 0 & \cos R_2 \end{pmatrix}
\]

\[
R_z = \begin{pmatrix} \cos R_3 & \sin R_3 & 0 \\ -\sin R_3 & \cos R_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]
Transformation between TRS (2/2)

In space geodesy we use the linearized formula:

\[ X_2 = X_1 + T + DX_1 + RX_1 \]

with: \( T = (T_x, T_y, T_z)^T \), \( \lambda = (1 + D) \), and \( R = (I + R) \)

where

\[
R = \begin{pmatrix}
  0 & -R_3 & R_2 \\
  R_3 & 0 & -R_1 \\
  -R_2 & R_1 & 0 \\
\end{pmatrix}
\]

since \( T \) is less than 100 meters, \( D \) & \( R \) less than \( 10^{-5} \)

The terms of 2nd order are neglected: less than \( 10^{-10} \approx 0.6 \text{ mm} \).

Differentiating equation 1 with respect to time, we have:

\[
\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + \ddot{D}X_1 + RX_1 + \ddot{R}X_1
\]
From one RF to another?

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}_{2} =
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}_{1} + 
\begin{pmatrix}
T_x \\
T_y \\
T_z
\end{pmatrix} + 
\begin{pmatrix}
D & -R_z & R_y \\
R_z & D & -R_x \\
-R_y & R_x & D
\end{pmatrix} 
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}_{1}
\]
Coordinate Systems

- Cartesian: X, Y, Z
- Ellipsoidal: $\lambda$, $\varphi$, h
- Mapping: E, N, h
- Spherical: R, $\theta$, $\lambda$
- Cylindrical: $l$, $\lambda$, Z

\[
\begin{align*}
\text{Cylindrical} & : OP \begin{bmatrix} l \cos \lambda \\ l \sin \lambda \\ z \end{bmatrix} \\
\text{Spherical} & : OP \begin{bmatrix} R \cos \theta \cos \lambda \\ R \cos \theta \sin \lambda \\ R \sin \theta \end{bmatrix}
\end{align*}
\]
Ellipsoidal and Cartesian Coordinates: Ellipsoid definition

- **a**: semi major axis
- **b**: semi minor axis
- **f**: flattening
- **e**: eccentricity

\[
e^2 = \frac{a^2 - b^2}{a^2}, \quad f = \frac{a - b}{a}
\]

(a,b), (a,f), or (a,e^2) define entirely and geometrically the ellipsoid
Ellipsoidal and Cartesian Coordinates

\[
\begin{align*}
X &= (N + h) \cos \lambda \cos \varphi \\
Y &= (N + h) \sin \lambda \cos \varphi \\
Z &= [N(1 - e^2) + h] \sin \varphi
\end{align*}
\]

\[
N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}
\]

Prime Vertical Radius of Curvature
(X, Y, Z) ==> (λ,ϕ,h)

\[ f = 1 - \sqrt{1 - e^2} \]
\[ R = \sqrt{X^2 + Y^2 + Z^2} \]
\[ \lambda = \arctg \left( \frac{Y}{X} \right) \]

\[ \mu = \arctg \left[ \frac{Z}{\sqrt{X^2 + Y^2}} \left( (1 - f) + \left( \frac{e^2 a}{R} \right) \right) \right] \]

\[ \varphi = \arctg \left[ \frac{Z(1 - f) + e^2 a \sin^3 \mu}{(1 - f) [X^2 + Y^2 - e^2 a \cos^3 \mu]} \right] \]

\[ h = \sqrt{X^2 + Y^2} \left[ \cos \varphi + Z \sin \varphi \right] - a \sqrt{1 - e^2 \sin^2 \varphi} \]
Map Projection

Mathematical function from an ellipsoid to a plane (map)

\[ E = f(\lambda, \varphi) \]
\[ N = g(\lambda, \varphi) \]

Mapping coordinates: (E,N,h)
Why a Reference System/Frame is needed?

• Precise Orbit Determination for:
  – GNSS: Global Navigation Satellite Systems
  – Other satellite missions: Altimetry, Oceanography, Gravity

• Earth Sciences Applications
  – Tectonic motion and crustal deformation
  – Mean sea level variations
  – Earth rotation
  – ...

• Geo-referencing applications
  – Navigation: Aviation, Terrestrial, Maritime
  – National geodetic systems
  – Cartography & Positioning
What is a Reference Frame?

• Earth fixed/centred RF: allows determination of station location/position as a function of time

• It seems so simple, but … we have to deal with:
  – Relativity theory
  – Forces acting on the satellite
  – The atmosphere
  – Earth rotation
  – Solid Earth and ocean tides
  – Tectonic motion
  – …

• Station positions and velocities are now determined with mm and mm/yr precision
"Motions" of the deformable Earth

- Nearly linear motion:
  - Tectonic motion: horizontal
  - Post-Glacical Rebound: Vertical & Horizontal

- Non-Linear motion:
  - Seasonal: Annual, Semi & Inter-Annual caused by loading effects
  - Rupture, transient: uneven motion caused by EQ, Volcano Eruptions, etc.
Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch $t$ could be written as:

$$X(t) = X_0 + \dot{X} \cdot (t - t_0) + \sum \Delta X_i(t)$$

$X_0$ : point position at a reference epoch $t_0$

$\dot{X}$ : point linear velocity

$\Delta X_i(t)$ : high frequency time variations:

- Solid Earth, Ocean & Pole tides
- Loading effects: atmosphere, ocean, hydrology, Post-glacial-Rebound
- ... Earthquakes
Reference Frame Representations

• "Quasi-Instantaneous" Frame: mean station positions at "short" interval:
  – One hour, 6-h, 12-h, one day, one week
  ==> Non-linear motion embedded in time series of instantaneous frames

• Long-Term Secular Frame: mean station positions at a reference epoch \((t_0)\) and station velocities: \(X(t) = X_0 + V^*(t - t_0)\)
Implementation of a TRF

- Definition at a chosen epoch, by selecting 7 parameters, tending to satisfy the theoretical definition of the corresponding TRS
- A law of time evolution, by selecting 7 rates of the 7 parameters, assuming linear station motion!
- ==> 14 parameters are needed to define a TRF
How to define the 14 parameters?
« TRF definition »

- **Origin & rate**: CoM (Satellite Techniques)
- **Scale & rate**: depends on physical parameters
- **Orientation**: conventional
- **Orient. Rate**: conventional: Geophysical meaning (Tectonic Plate Motion)

- ==> Lack of information for some parameters:
  - Orientation & rate (all techniques)
  - Origin & rate in case of VLBI

    ==> Rank Deficiency in terms of Normal Eq. System
Implmentation of a TRF in practice

The normal equation constructed upon observations of space techniques is written in the form of:

\[ N.(\Delta X) = K \]  \hspace{1cm} (1)

where \( \Delta X = X_{est} - X_{apr} \) are the linearized unknowns

Eq. (1) is a singular system: has a rank deficiency equal to the number of TRF parameters not given by the observations. Additional constraints are needed:

- **Tight constraints** \( (\sigma \leq 10^{-10}) \text{ m} \) \hspace{1cm} Applied over station coordinates
- **Removable constraints** \( (\sigma \approx 10^{-5}) \text{ m} \)
- **Loose constraints** \( (\sigma \geq 1) \text{ m} \)

- **Minimum constraints** (applied over the TRF parameters, see next)
TRF definition using minimum constraints (1/3)

The standard relation linking two TRFs 1 and 2 is:

\[ X_2 = X_1 + A\theta \]

\[ X_i = (x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i)^T \]

\[ \theta = (T_1, T_2, T_3, D, R_1, R_2, R_3, \dot{T}_1, \dot{T}_2, \dot{T}_3, \dot{D}, \dot{R}_1, \dot{R}_2, \dot{R}_3)^T \]

\( \theta \) is the vector of the 7 (14) transformation parameters

Least squares adjustment gives for \( \theta \):

\[ \theta = (A^T A)^{-1} A^T (X_2 - X_1) \]

\( A \) : design matrix of partial derivatives given in the next slide
The Design matrix $A$

14 parameters

7 parameters

$$A = \begin{pmatrix} 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\ 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\ 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \approx 0$$

Note: $A$ could be reduced to specific parameters. E.g. if only rotations and rotation rates are needed, then the first 4 columns of the two parts of $A$ are deleted.
TRF definition using minimum constraints (2/3)

- The equation of minimum constraints is written as:

$$B(X_2 - X_1) = 0 \quad (\Sigma_\theta)$$

It nullifies the 7 (14) transformation parameters between TRF 1 and TRF 2 at the $\Sigma_\theta$ level

- The normal equation form is written as:

$$B^T \Sigma_\theta^{-1} B(X_2 - X_1) = 0$$

$\Sigma_\theta$ is a diagonal matrix containing small variances of the 7(14) parameters, usually at the level of 0.1 mm
TRF definition using minimum constraints (3/3)

Considering the normal equation of space geodesy:

\[ N_{nc}(\Delta X) = K \]  \hspace{1cm} (1)

where \( \Delta X = X_{est} - X_{apr} \) are the linearized unknowns.

Selecting a reference solution \( X_R \), the equation of minimal constraints is given by:

\[ B^T \Sigma^{-1}_\theta B(\Delta X) = B^T \Sigma^{-1}_\theta B(X_R - X_{apr}) \]  \hspace{1cm} (2)

Accumulating (1) and (2), we have:

\[ (N_{nc} + B^T \Sigma^{-1}_\theta B)(\Delta X) = K + B^T \Sigma^{-1}_\theta B(X_R - X_{apr}) \]

Note: if \( X_R = X_{apr} \), the 2nd term of the right-hand side vanishes.
Combination of daily or weekly TRF solutions (1/3)

The basic combination model is written as:

\[ X^i_s = X^i_c + T_s + D_s X^i_c + R_s X^i_c \]

Inputs: \( X^i_s \), coordinates of point \( i \) of individual solution \( s \).

Outputs (unknowns): combined coordinates \( X^i_c \) and transformation parameters \( T_s, D_s, R_s \) from TRF \( s \) to TRF \( c \).

Note that the translation vector \( T_s \) and the rotation matrix \( R_s \) have each three components around the three axes \( X, Y, Z \).

The unknown parameters are linearized around their approximate values: \( x^i_0, y^i_0, z^i_0 \), so that \( x^i_c = x^i_0 + \delta x^i \) (respectively \( y^i_c, z^i_c \)).

Note: this combination model is valid at a give epoch, \( t_s \), both for the input and output station coordinates.
Combination of daily or weekly TRF solutions (2/3)

The observation equation system is written as:

\[
\begin{pmatrix} I & A_s \end{pmatrix} \begin{pmatrix} \delta \chi_s \\ \delta T_s \end{pmatrix} + B_s = V_s
\]

and the normal equation is:

\[
\begin{pmatrix} P_s & P_s A_s \\ A_s^T P_s & A_s^T P_s A_s \end{pmatrix} \begin{pmatrix} \delta \chi_s \\ \delta T_s \end{pmatrix} + \begin{pmatrix} P_s B_s \\ A_s^T P_s B_s \end{pmatrix} = 0
\]

where \( I \) is the identity matrix, \( A_s \) is the design matrix related to solution \( s \), \( \delta \chi_s \) and \( \delta T_s \) are the linearized unknowns of station coordinates and transformation parameters, respectively. \( B_s \) are the (observed - computed) values and \( V_s \) are the residuals. \( P_s \): weight matrix = \( \Sigma_s^{-1} \): inverse of variance-covariance matrix.
Combination of daily or weekly TRF solutions (3/3)

The design matrixs $A_s$ has the following form:

$$A_s = \begin{pmatrix}
1 & 0 & 0 & x^i_0 & 0 & z^i_0 & -y^i_0 \\
0 & 1 & 0 & y^i_0 & -z^i_0 & 0 & x^i_0 \\
0 & 0 & 1 & z^i_0 & y^i_0 & -x^i_0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{pmatrix}$$
Definition of the combined TRF

- The normal equation system described in the previous slides is singular and has a rank deficiency of 7 parameters.
- The 7 parameters are the defining parameters of the combined TRF $c$: origin (3 components), scale (1 component) and orientation (3 components).
- The combined TRF $c$, could be defined by, e.g.:
  - Fixing to given values 7 parameters among those to be estimated
  - Using minimum constraint equation over a selected set of stations of a reference TRF solution $X_R$. Refer to slide 24 for more details…
Combination of long-term TRF solutions

The basic combination model is extended to include station velocities and is written as:

\[
\begin{align*}
X^i_s &= X^i_c + T_s + D_s X^i_c + R_s X^i_c \\
\dot{X}^i_s &= \dot{X}^i_c + \dot{T}_s + \dot{D}_s X^i_c + \dot{R}_s X^i_c
\end{align*}
\]

where the dotted parameters are their time derivatives.

Inputs: \(X^i_s\), position of point \(i\), at epoch \(t_s\) and velocities, \(\dot{X}^i_s\), of individual solution \(s\).

Outputs: combined positions \(X^i_c\), at epoch \(t_s\), velocities and transformation parameters \(T_s, D_s, R_s\), at epoch \(t_s\), from TRF \(s\) to TRF \(c\).

In the same way as for daily or weekly TRF combination, observation and normal equations could easily be derived.

Note: this combination model is only valid at a give epoch, both for the input and output station coordinates.
Stacking of TRF time series

The basic combination model is written as:

\[ X_s^i = X_c^i(t_0) + (t_s - t_0)\dot{X} + T_s + D_s X_c^i + R_s X_c^i \]

Inputs: Time series of station positions, \( X_s^i \), at different epochs \( t_s \).

Outputs: combined positions \( X_c^i \) at epoch \( t_0 \), velocities and transformation parameters \( T_s, D_s, R_s \) from TRF \( s \) to TRF \( c \).

Here also, observation and normal equations are constructed and solved by least squares adjustment.
Space Geodesy Techniques

- Very Long Baseline Interferometry (VLBI)
- Lunar Laser Ranging (LLR)
- Satellite Laser Ranging (SLR)
- DORIS
- GNSS: GPS, GLONASS, GALILEO, COMPASS, ...

- Local tie vectors at co-location sites
Complex of Space Geodesy instruments

- SLR/LLR
- VLBI
- GPS
- DORIS
## Reference frame definition by individual techniques

<table>
<thead>
<tr>
<th></th>
<th>Satellite Techniques</th>
<th>VLBI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Origin</strong></td>
<td>Center of Mass</td>
<td>-</td>
</tr>
<tr>
<td><strong>Scale</strong></td>
<td>GM, c &amp; Relativistic corrections</td>
<td>c Relativistic corrections</td>
</tr>
<tr>
<td><strong>Orientation</strong></td>
<td>Conventional</td>
<td>Conventional</td>
</tr>
</tbody>
</table>
Current networks: stations observed in 2011

**VLBI**

**SLR**

**GPS/IGS**

**DORIS**
International Association of Geodesy
International Services

- Intern. GNSS Service (IGS) (1994)
- Intern. VLBI Service (IVS) (1999)

http://www.iag-aig.org/
International Terrestrial Reference System (ITRS)

Realized and maintained by the IERS
International Earth Rotation and Reference Systems Service (IERS)

Established in 1987 (started Jan. 1, 1988) by IAU and IUGG to realize/maintain/provide:

- The International Celestial Reference System (ICRS)
- The International Terrestrial Reference System (ITRS)
- Earth Orientation Parameters (EOP)
- Geophysical data to interpret time/space variations in the ICRF, ITRF & EOP
- Standards, constants and models (i.e., conventions)

http://www.iers.org/
International Terrestrial Reference System (ITRS): Definition (IERS Conventions)

• **Origin**: Center of mass of the whole Earth, including oceans and atmosphere

• **Unit of length**: meter SI, consistent with TCG (Geocentric Coordinate Time)

• **Orientation**: consistent with BIH (Bureau International de l’Heure) orientation at 1984.0.

• **Orientation time evolution**: ensured by using a No-Net-Rotation-Condition w.r.t. horizontal tectonic motions over the whole Earth

\[ h = \int_C X \times V \, dm = 0 \]
International Terrestrial Reference System (ITRS)

- Realized and maintained by ITRS Product Center of the IERS
- Its Realization is called International Terrestrial Reference Frame (ITRF)
- Set of station positions and velocities, estimated by combination of VLBI, SLR, GPS and DORIS individual TRF solutions
- Based on Co-location sites

Adopted by IUGG in 1991 for all Earth Science Applications

More than 800 stations located on more than 500 sites

Available: ITRF88, ..., 2000, 2005
Latest: ITRF2008

http://itrf.ign.fr
Co-location site

- Site where two or more instruments are operating
- Surveyed in three dimensions, using classical or GPS geodesy
- Differential coordinates (DX, DY, DZ) are available

\[ DX_{\text{(GPS,VLBI)}} = X_{\text{VLBI}} - X_{\text{GPS}} \]
**Strengths:**

**Contribution of Geodetic Techniques to the ITRF**

<table>
<thead>
<tr>
<th>Technique Signal Source Obs. Type</th>
<th>VLBI Microwave Quasars Time difference</th>
<th>SLR Optical Satellite Two-way absolute range</th>
<th>GPS Microwave Satellites Range change</th>
<th>DORIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celestial Frame &amp; UT1</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Polar Motion</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Scale</td>
<td>Yes</td>
<td>Yes</td>
<td>No (but maybe in the future!)</td>
<td>Yes</td>
</tr>
<tr>
<td>Geocenter ITRF Origin</td>
<td>No</td>
<td>Yes</td>
<td>Future</td>
<td>Future</td>
</tr>
<tr>
<td>Geographic Density</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Real-time &amp; ITRF access</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Decadal Stability</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Mix of techniques is fundamental to realize a frame that is stable in origin, scale, and with sufficient coverage.
How the ITRF is constructed?

**Input:**
- Time series of mean station positions (at weekly or daily sampling) and daily EOPs from the 4 techniques
- Local ties in co-location sites

**Output:**
- Station positions at a reference epoch and linear velocities
- Earth Orientation Parameters

CATREF combination model

\[
\begin{aligned}
X^i_s &= X^i_c + (t^i_s - t_0) \dot{X}^i_c \\
&\quad + T_k + D_k X^i_c + R_k X^i_c \\
&\quad + (t^i_s - t_k) \left[ \dot{T}_k + \dot{D}_k X^i_c + \dot{R}_k X^i_c \right]
\end{aligned}
\]

\[
\begin{aligned}
\dot{X}^i_s &= \dot{X}^i_c + \dot{T}_k + \dot{D}_k X^i_c + \dot{R}_k X^i_c
\end{aligned}
\]

\[
\begin{aligned}
x^p_s &= x^p_c + R2_k \\
y^p_s &= y^p_c + R1_k \\
UT_s &= UT_c - \frac{1}{f} R3_k \\
\dot{x}^p_s &= \dot{x}^p_c \\
\dot{y}^p_s &= \dot{y}^p_c \\
LOD_s &= LOD_c
\end{aligned}
\]
ITRF Construction

Local ties

Velocity equality

At co-location sites

Time series stacking

Combination

ITRF Solution

Long-term Solutions

DORIS
GPS
SLR
VLBI

X

X

DORIS
GPS
SLR
VLBI
Power of station position time series

- Monitor station behavior
  - Linear, non-linear (seasonal), and discontinuities

- Monitor time evolution of the frame physical parameter (origin and scale)

- Estimate a robust long-term secular frame
Some examples of discontinuities and seasonal variations
Dicontinuity due to equipment change

Before

After

EUSK 14258 M003 Residuals (mm)

NORTH

EAST

UP

1998 1999 2000 2001 2002 2003

1998 1999 2000 2001 2002 2003

1998 1999 2000 2001 2002 2003

1998 1999 2000 2001 2002 2003

Technical Seminar on Reference Frame in Practice Rome - Italy, 4th-5th May 2012
Denaly Earthquake (Alaska)
Arequipa Earthquake

Arequipa GPS Point

Arequipa SLR Point
Example of seasonal variations
BRAZ GPS antenna
ITRF2008

• Time Series of Station Positions:
  – Daily (VLBI)
  – Weekly (GPS, SLR & DORIS)

• and Earth Orientation Parameters:
  Polar Motion \((x_p, y_p)\)
  Universal Time (UT1) (Only from VLBI)
  Length of Day (LOD) (Only from VLBI)
ITRF2008 Network

580 sites (920 stations)

461 Sites North

118 Sites South
ITRF2008 Datum Specification

- Origin: SLR
- Scale: Mean of SLR & VLBI
- Orientation: Aligned to ITRF2005 using 179 stations located at 131 sites:
  104 at northern hemisphere and 27 at southern hemisphere
SLR & DORIS origin components wrt ITRF2008

SLR

DORIS
Scales wrt ITRF2008
How to estimate an absolute plate rotation pole?

\[ \dot{X} = \omega_p \times X \]

- TRF definition
- Number and distribution over sites over the plate
- Quality of the implied velocities
- Level of rigidity of the plate
Plate boundaries: Bird (2003) and MORVEL, DeMets et al. (2010)
ALL ITRF2008 Site Velocities:

time-span > 3 yrs

509 sites
Selected Site Velocities

Plate angular velocity \( \omega_p \) is estimated by:

\[ \dot{X}_i = \omega_p \times X_i \]

213 sites
Comparison between ITRF2008 & NNR-NUVEL-1 & NNR-MORVEL56

After rotation rate transformation

NNR-NUVEL-1A
RMS:
East: 2.4 mm/yr
North: 2.1 mm/yr

Green: < 2 mm/yr
Blue: 2-3 mm/yr
Orange: 3-4 mm/yr
Red: 4-5 mm/yr
Black: > 5 mm/yr

NNR-MORVEL56
RMS:
East: 1.8 mm/yr
North: 1.9 mm/yr
Plate motion and Glacial Isostatic Adjustment

Blue: points used
Red: points rejected

Residual velocities after removing NOAM & EURA rotation poles
ITRF2008 Vertical velocity field
## ITRF transformation parameters

Table 4.1: Transformation parameters from ITRF2008 to past ITRFs. “ppb” refers to parts per billion (or $10^{-9}$). The units for rates are understood to be “per year.”

<table>
<thead>
<tr>
<th>ITRF</th>
<th>Solution</th>
<th>$T1$ (mm)</th>
<th>$T2$ (mm)</th>
<th>$T3$ (mm)</th>
<th>$D$ (ppb)</th>
<th>$R1$ (mas)</th>
<th>$R2$ (mas)</th>
<th>$R3$ (mas)</th>
<th>Epoch</th>
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<tr>
<td>ITRF2005 rates</td>
<td>-2.0</td>
<td>-0.9</td>
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<td>0.00</td>
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<td>2.92</td>
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<td>0.00</td>
<td>2000.0</td>
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</tbody>
</table>
Access & alignment to ITRF

• Direct use of ITRF coordinates
• Use of IGS Products (Orbits, Clocks): all expressed in ITRF

• Alternatively:
  – Process GNSS data together with IGS/ITRF global stations in free mode
  – Align to ITRF by
    • Constraining station coordinates to ITRF values at the central epoch of the observations
    • Using minimum constraints approach
Transformation from an ITRF to another at epoch $t_c$

- **Input:** $X$ (ITRFxx, epoch $t_c$)
- **Output:** $X$ (ITRFyy, epoch $t_c$)
- **Procedure:**
  - Propagate ITRF transformation parameters from their epoch (2000.0, slide 64) to epoch $t_c$, for both ITRFxx and ITRFyy:
    \[
    P(t_c) = P(2000.0) + \dot{P}(t_c - 2000.0)
    \]
  - Compute the transformation parameters between ITRFxx and ITRFyy, by subtraction;
  - Transform using the general transformation formula given at slide 10:
    \[
    X(\text{ITRFyy}) = X(\text{ITRFxx}) + T + D.X(\text{ITRFxx}) + R.X(\text{ITRFxx})
    \]
How to express a GPS network in the ITRF?

- Select a reference set of ITRF/IGS stations and collect RINEX data from IGS data centers;
- Process your stations together with the selected ITRF/IGS ones:
  - Fix IGS orbits, clocks and EOPs
  - Eventually, add minimum constraints conditions in the processing

$\Rightarrow$ Solution will be expressed in the ITRFyy consistent with IGS orbits

- Propagate official ITRF station positions at the central epoch ($t_c$) of the observations:
  \[ X(t_c) = X(t_0) + \dot{X}(t_c - t_0) \]
- Compare your estimated ITRF station positions to official ITRF values and check for consistency!
From the ITRF to Regional Reference Frames

- **Purpose:** geo-referencing applications ($\sigma \sim$ cm)
- **There are mainly two cases/options to materialize a regional reference frame:**
  1. Station positions at a given epoch, eventually updated frequently. Ex.: North & South Americas
  2. Station positions & minimized velocities or station positions & deformation model. Ex.: Europe (ETRS89) New Zealand, Greece (?)
    - Case 1 is easy to implement (see previous slide)
    - Case 2 is more sophisticated & needs application of:
      - Transformation formula (ETRS89)
      - Deformation model
## GNSS and their associated reference systems

<table>
<thead>
<tr>
<th>GNSS</th>
<th>Ref. System/Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS (broadcast orbits)</td>
<td>WGS84</td>
</tr>
<tr>
<td>GPS (precise IGS orbits)</td>
<td>ITRS/ITRF</td>
</tr>
<tr>
<td>GLONASS</td>
<td>PZ-90</td>
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<tr>
<td>GALILEO</td>
<td>ITRS/ITRF/GTRF</td>
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<tr>
<td>COMPASS</td>
<td>CGCS 2000</td>
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<tr>
<td>QZSS</td>
<td>JGS</td>
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</table>

- All are "aligned" to the ITRF
- WGS84 $\approx$ ITRF at the decimeter level
- GTRF $\approx$ ITRF at the mm level
- $\sigma$-Position using broadcast ephemerides $= 150$ cm
The World Geodetic System 84 (WGS 84)

• Collection of models including Earth Gravitational model, geoid, transformation formulae and set of coordinates of permanent DoD GPS monitor stations

• WGS 60…66…72…84

• Originally based on TRANSIT satellite DOPPLER data
The World Geodetic System 84 (WGS 84)

- Recent WGS 84 realizations based on GPS data:
  - G730 in 1994
  - G873 in 1997
  - G1150 in 2002
- Coincides with any ITRF at 10 cm level
- No official Transf. Param. With ITRF
WGS 84-(G1150)
WGS 84-(G1150)

- Coordinates of ~20 stations fixed to ITRF2000
- No station velocities
WGS84 - NGA Stations in ITRF2008
NGA: National Geospatial-Intelligence Agency

NGA stations in ITRF2008
WGS84 - NGA Stations in ITRF2008

Horizontal Velocities of NGA stations

Major plate boundaries are shown in green

2 cm/y

Zuhair Altamimi
**Galileo Terrestrial Reference Frame (GTRF)**

- **Galileo Geodesy Service Provider (GGSP)**

- **GGSP Consortium (GFZ, AIUB, ESOC, BKG, IGN)**
  - Define, realize & maintain the GTRF
  - GTRF should be "compatible" with the ITRF at 3 cm level
  - Liaison with IERS, IGS, ILRS

- **GTRF is a realization of the ITRS**
The GTRF Experience

- Initial GSS positions & velocities are determined using GPS observations
- Subsequent GTRF versions using GPS & Galileo observations
- Ultimately Galileo Observations only
GTRF09v01 horizontal velocities
### Comparison of GTRF09v01 to ITRF2005

#### Transformation parameters

<table>
<thead>
<tr>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>D</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
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<td>mm</td>
<td>mm</td>
<td>10⁻⁹</td>
<td>mas</td>
<td>mas</td>
<td>mas</td>
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<td>0.2</td>
<td>0.03</td>
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Rates:

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<th>T2</th>
<th>T3</th>
<th>D</th>
<th>R1</th>
<th>R2</th>
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<tr>
<td>mm</td>
<td>mm</td>
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<td>10⁻⁹</td>
<td>mas</td>
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<td>mas</td>
<td>y</td>
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<td>0.03</td>
<td>0.007</td>
<td>0.008</td>
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### RMS difference between stations coordinates and velocities

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<th>WRMS-Vel.</th>
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<tr>
<td></td>
<td>E</td>
<td>N</td>
<td>U</td>
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<tr>
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=> Perfect GTRF alignment to the ITRF at the sub-mm level
Conclusion (1/2)

- The ITRF
  - is the most optimal global RF available today
  - gathers the strengths of space geodesy techniques
  - more precise and accurate than any individual RF

- Using the ITRF as a common GNSS RF will facilitate the interoperability

- Well established procedure available to ensure optimal alignment of GNSS RFs to ITRF

- To my knowledge: most (if not all) GNSS RFs are already ‘’aligned’’ to ITRF

- GNSS RFs should take into account station velocities
Conclusion (2/2)

WGS84, PZ90, GTRF
Are all connected to (compatible with)
a Unique System
The ITRS