Local Anisotropy in the Parker's Solar Dynamo Model

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MHD-dynamo theory

The MHD-dynamo theory describes the processes of generation and evolution of an average magnetic field in random turbulent flows. Dynamo theory is traditionally divided into large-scale mean-field dynamo and small-scale turbulent dynamo. These models are based on two different assumptions. We propose a method that allows us to derive both models on the one same assumption.

The method of multiplicative integrals works for short-correlated flows, where magnetic fields are correlated at times much more than velocity fields, which makes it possible to average over a random velocity field. The multiplicative approach implies the reduction of the induction equation to the form of a multiplicative integral. We take the ideal induction equation:

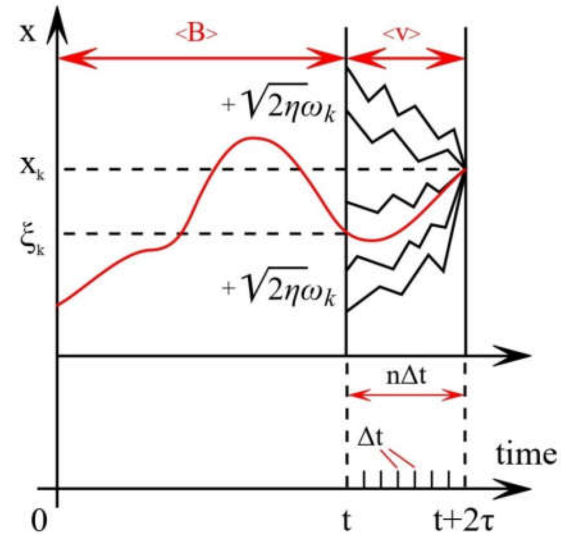
$$\boxed{d_t(\mathbf{A}) = [\mathbf{v}, \mathrm{rot}(\mathbf{A})]}$$

The using of a vector potential in this method has several advantages and facilitates the derivation of equations in general.

Multiplicative integral

$$\boxed{d_t(\mathbf{A}) = [\mathbf{v}, \mathrm{rot}(\mathbf{A})]} \longrightarrow (\mathrm{A_i})_\mathrm{t} + \mathrm{v_j}\nabla_\mathrm{j}\mathrm{A_i} = -\mathrm{A_j}\nabla_\mathrm{i}\mathrm{v_j} + \nabla_\mathrm{i}(\mathrm{v_j}\mathrm{A_j})$$

$$d_t(A_i) = -A_j \nabla_i v_j$$



The total derivative allows us to write the change of the vector potential in a liquid particle of the medium that has moved from the starting point $\xi(t)$ to the point $x(t+\Delta t)$ in time Δt :

$$A_i(x, t+\Delta t) = A_i(\xi, t) + d_t A_i(\xi, t)\Delta t = (A_i - A_j \nabla_i v_j \Delta t)(\xi, t) = (\delta_{ij} - \nabla_i v_j \Delta t) A_j(\xi, t)$$

For an arbitrary point in time $\Delta = n\Delta t$

$$\boxed{A_i(x, t+\Delta) = \prod_1^n (\delta_{ij} - \nabla_i v_j(x(s), s)\Delta t) A_j(\xi, t)}$$ [Manturov, 1990]

To take into account magnetic diffusion, we should replace the deterministic trajectory of a liquid particle with a bunch of random Wiener trajectories

$$\xi_k = x_k - \int_t^{t+\Delta} v_k(x(s), s)ds + \boxed{\sqrt{2\eta} w_k,}$$

Asymptotically rewrite the right side at the end point of the trajectory

$$A_i(x,t+\Delta) = \left(\delta_{ij} + \sum_1^n(-\nabla_i v_j \Delta t) + \frac{1}{2}\sum_1^n\sum_1^n \nabla_i v_k \nabla_k v_j \Delta t^2 + \ldots\right) A_j(\xi,t)$$

The multiplicative integral and the vector potential are expanded:

$$A_j(\xi,t) = A_j(x,t) + \nabla_k A_j \boxed{(\xi_k - x_k)} + \frac{1}{2}\nabla_k\nabla_l A_j(\xi_k - x_k)(\xi_l - x_l) + ..$$

$$\boxed{(\xi_k - x_k)} = \sqrt{2\eta}w_k - v_k(x,t)\Delta - \nabla_l v_k(x,t)\sqrt{2\eta}w_l\Delta + v_l\nabla_l v_k(x,t)\frac{\Delta^2}{2} + \ldots$$

And after averaging over the Wiener process, we obtain:

$$A_i(x,t+\Delta) = A_i(x,t) - v_j\nabla_j A_i\Delta - \nabla_i v_j A_j\Delta + \left(\eta\delta_{kl}\Delta + v_k v_l\frac{\Delta^2}{2}\right)\nabla_k\nabla_l A_i +$$

$$v_l\nabla_l v_k\nabla_k A_i\frac{\Delta^2}{2} + v_k\nabla_i v_j\nabla_k A_j\Delta^2 + v_k\nabla_k\nabla_i v_j\frac{\Delta^2}{2} + \nabla_k v_j\nabla_i v_k A_j\frac{\Delta^2}{2}$$

In the case of a deterministic speed, we get the induction equation

$$\boxed{\mathbf{A}_t = [\mathbf{v}, \mathrm{rot}(\mathbf{A})] + \eta\Delta\mathbf{A}}$$

When averaging over the random velocity field and over the vector potential, we obtain the mean field equation

$$\boxed{\mathbf{B}_t = \mathrm{rot}[\mathbf{V},\mathbf{B}] + \mathrm{rot}(\alpha\cdot\mathbf{B}) - \mathrm{rot}(\mathrm{rot}(\beta\cdot\mathbf{B}))}$$

$$\alpha = -\frac{\Delta}{6}\langle(v, \mathrm{rot(v)})\rangle \longrightarrow \frac{1}{2}\langle v_k\nabla_i v_j - v_j\nabla_i v_k\rangle = -\alpha\varepsilon_{ijk}$$

$$\beta = \eta + \frac{\Delta}{6}\langle(v,v)\rangle \longrightarrow \beta_{kl} = \eta\delta_{kl} + \langle v_k v_l\rangle/2$$

Locally anisotropic Parker dynamo in a spherical coordinate system

So-called generalized Steenbeck-Krause-Radler equation has the form:

$$\boxed{\dot{B}_i = \varepsilon_{inm}\nabla_n\left(\varepsilon_{mjk}(V_j - W_j)B_k + \alpha_{mk}B_k - \varepsilon_{mjk}\beta_{lj}\nabla_l B_k\right)}$$

In this paper, we consider the effects associated with axial anisotropy, that is, we will consider the equation

$$\dot{\mathbf{B}} = \mathrm{rot}[\mathbf{V}, \mathbf{B}] + \mathrm{rot}\{\alpha B_x; \alpha B_y, \overline{\alpha} B_z\} + (\beta\partial_x^2 + \beta\partial_y^2 + \overline{\beta}\partial_z^2)\mathbf{B}$$

We divide the field into toroidal and poloidal components

$$\mathbf{B} = \mathrm{rot}(A\mathbf{e}_\varphi) + B\mathbf{e}_\varphi = \left(\frac{\partial_\theta(A\sin\theta)}{r\sin\theta}\,;\,-\frac{\partial_r(Ar)}{r}\,;\,B\right)$$

In the case of azimuthal symmetry, we obtain the standard Parker system

$$\dot{A} = \alpha B + \beta r^{-1}\partial_r^2(Ar) + \beta r^{-2}\partial_\theta(\sin^{-1}\theta\partial_\theta(A\sin\theta)) + D^A_{anis}$$

$$\begin{aligned}\dot{B} = {} & \partial_r\Omega r^{-1}\partial_\theta(Ar\sin\theta) - \partial_\theta\Omega r^{-1}\partial_r(Ar\sin\theta) - \\ & -\alpha r^{-1}\partial_{rr}^2(Ar) - \alpha r^{-1}\partial_\theta(r^{-1}\sin^{-1}\theta\partial_\theta(A\sin\theta)) + R_{anis} + \\ & +\beta r^{-1}\partial_r^2(Br) + \beta r^{-2}\partial_\theta(\sin^{-1}\theta\partial_\theta(B\sin\theta)) + D^B_{anis}\end{aligned}$$

$$R_{anis} = (\overline{\alpha} - \alpha)\mathrm{rot}(0, 0, B_z)$$

$$D^A_{anis} = (\overline{\beta} - \beta)\partial_z^2(A_x, A_y, A_z)$$

$$D^B_{anis} = (\overline{\beta} - \beta)\partial_z^2(B_x, B_y, B_z)$$

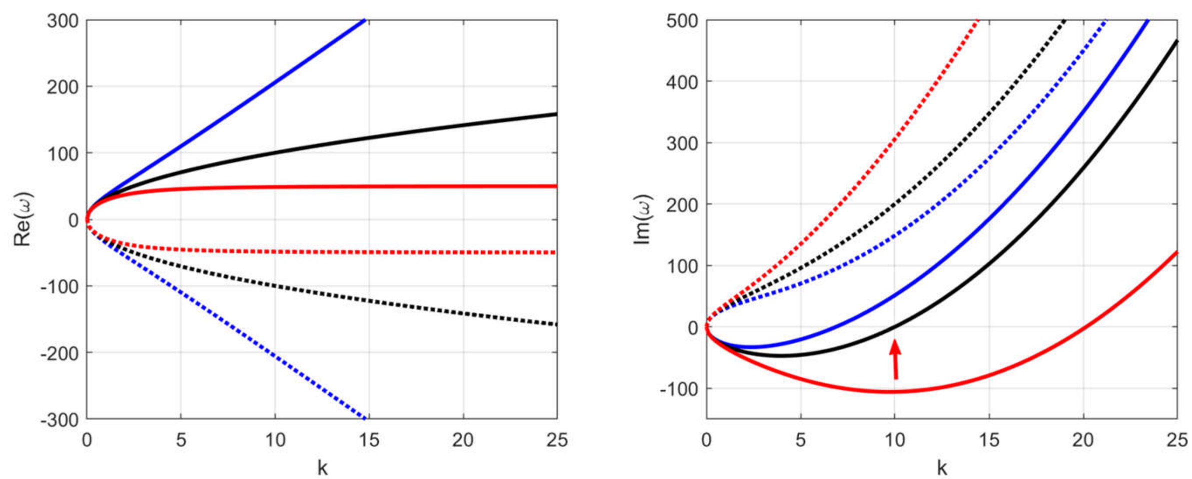
We neglect the derivative with respect to theta and get:

$$\dot{A} = R_\alpha B + \beta_{ef} A_{\theta\theta}, \qquad \beta_{ef} = (\overline{\beta}\sin^2\theta + \beta\cos^2\theta)/\overline{\beta}$$
$$\dot{B} = R_\omega A_\theta - R_\alpha \alpha_{ef} A_{\theta\theta} + \beta_{ef} B_{\theta\theta}, \qquad \alpha_{ef} = (\overline{\alpha}\cos^2\theta + \alpha\sin^2\theta)/\alpha$$

We are looking for a solution in the form of $\sim \exp(i\omega t + ik\theta)$ and obtain the dispersion equation

$$\boxed{\omega = ik^2\beta_{ef} \pm \sqrt{-ikR_\alpha R_\omega - k^2 R_\alpha^2 \alpha_{ef}}}$$

$$R_\alpha = \frac{\alpha R}{\overline{\beta}} \quad \text{и} \quad R_\omega = \frac{R^2}{\overline{\beta}} \frac{\partial_r \Omega}{r} \sin\theta$$



Real and imaginary parts of the Parker wave dynamo frequency for the isotropic case with $\alpha_{ef} = 1$ (red lines) and anisotropic cases with $\alpha_{ef} = 0$ and $\alpha_{ef} = -1$ (black and blue lines respectively). Solid and dashed lines correspond to different branches of the dispersion equation, $R_\omega = 100$, $R_\alpha = 20$, $\beta_{ef} = 1$.

Conclusions

- In the work we use method of multiplicative integrals. The method makes it possible to obtain the standard equations, the mean field equation, and the equation for the second moments using just one assumption. We derive these equations using a vector potential, because of the advantages that this approach gives us.
- The anisotropic analog of the mean field equation is studied, for this we replaced the hydrodynamic helicity and turbulent diffusion by the corresponding tensors.

$$\boxed{\dot{B}_i = \varepsilon_{inm} \nabla_n \left(\varepsilon_{mjk} (V_j - W_j) B_k + \alpha_{mk} B_k - \varepsilon_{mjk} \beta_{lj} \nabla_l B_k \right)}$$

- Taking into account even the simplest azimuthal anisotropy can significantly change the diffusion terms that are responsible for the characteristics of the generation, in particular, the dispersion law and for the position of the generation threshold from which this generation starts.
- Whether or not real local turbulence anisotropy is present in certain cases, for example, in a solar dynamo, and what role it plays is a further task of research. We have obtained a basic model for local turbulence anisotropy.