Small-scale magnetic helicity in short-correlated plasma turbulence

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Is there any influence of small-scale dynamo on the large-scale MHD processes?

The dynamo theory describes the process of magnetic fields generation in a random conducting media. Traditionally, such a process is conditionally divided into a **mean field dynamo** and a **turbulent dynamo**. What is the difference?

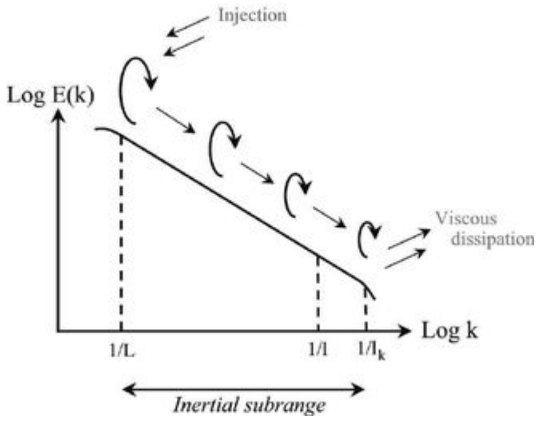
Big whirls have little whirls
That feed on their velocity,
And little whirls have lesser whirls
and so on to viscosity
[Lewis F. Richardson, 1922]

turbulent dynamo

- **only energy grows**
- **growth Rm-threshold**
- **small scales**
- **poorly studied**

mean field dynamo

- **averaged m.field grows**
- **helicity and dif.rotation**
- **large scales**
- **well studied**



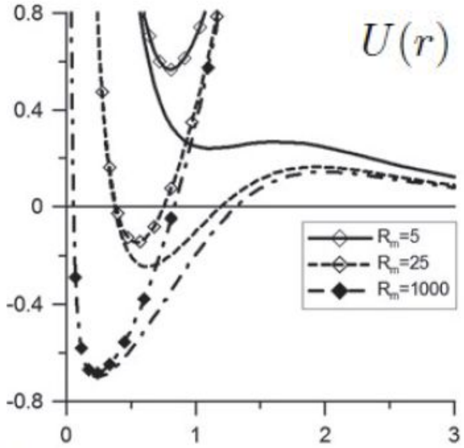
If we consider three-dimensional MHD turbulence in which only a direct energy cascade is supposed, does this mean that the influence of small-scale processes on large-scale one can be neglected?

and from dissipative to energy-bearing scales?

Small-scale turbulent dynamo model.

[Yushkov, GAFD, 2015]

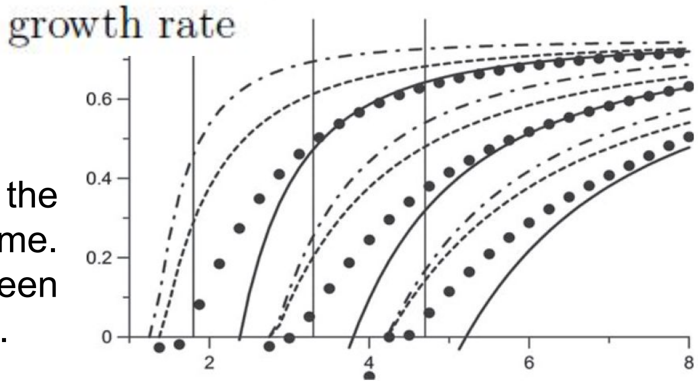
The basic model of a small-scale dynamo was proposed in 1967 by Kazantsev for a mirror-symmetrical uniform turbulence. Using the diagram technique, he connected the mean magnetic field correlation tensor and the velocity correlation tensor. To use the diagram technique, he considered a kinetic regime with a delta-time-correlated velocity field.



$$\langle B_i(r_1,t_1)B_j(r_2,t_2)\rangle = \left(M + \frac{r}{2}M_r\right)\delta_{ij} - \frac{M_r}{2r}r_i r_j$$

$$\psi(r,t) = r^2\eta^{1/2}M(r,t)$$

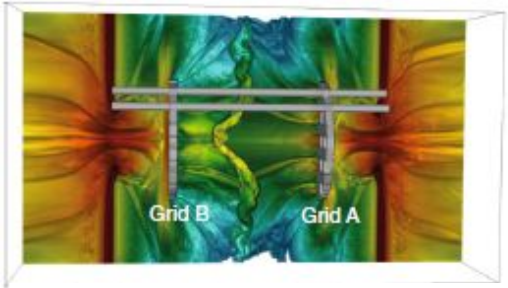
$$\frac{1}{2}\psi_t = \eta(r)\psi_{rr} - U(r)\psi$$



The main result: for large magnetic Reynolds numbers, the Kazantsev equation has a solution growing exponentially in time. The generation threshold and eigenfunctions have been repeatedly studied by both numerical and asymptotic methods.

VK system for small-scale dynamo

[Tzeferacos, NatCom, 2019]



It's clear that Kazantsev model has some obvious problems:

- **hard to be experimentally checked**
- **correlation times equal for all k**
- **strict mirror symmetry**

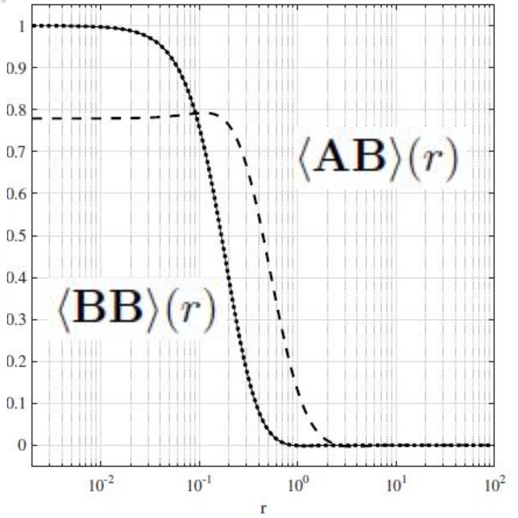
The problem of mirror symmetry was solved by Vainshtein and Kichatinov in 1986 by introducing the antisymmetric part of the correlation tensor:

$$\langle B_i(r_1,t_1)B_j(r_2,t_2)\rangle = \left(M + \frac{r}{2}M_r\right)\delta_{ij} - \frac{M_r}{2r}r_i r_j + K\varepsilon_{ijk}r_k$$

$$M_t = 2r^{-4}(r^4\eta M_r)_r + 2Mr^{-4}(r^4\eta_r)_r - 4\alpha K$$

$$K_t = r^{-4}(r^4(\alpha M + 2\eta K)_r)_r$$

$$\alpha = \tau(G(0) - G(r)) \qquad \eta = \frac{1}{Rm} + \frac{\tau}{3}(F(0) - F(r))$$

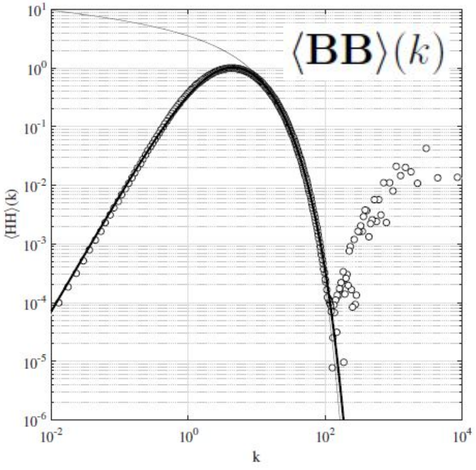


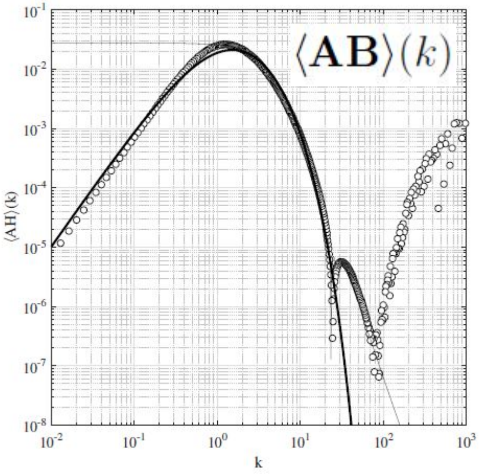
[Yushkov, GAFD, 2018]

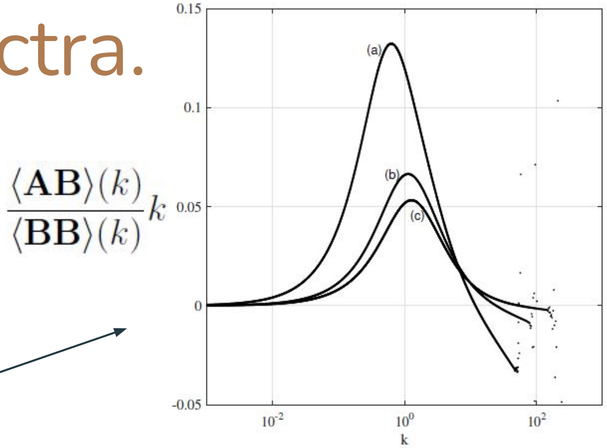
Small-scale helicity/energy spectra.

Fourier transform makes it possible to obtain expressions for the spectral densities from the correlation functions:

[Yushkov, GAFD, 2019]







$$\langle \boldsymbol{HH}\rangle(k) = \frac{1}{(2\pi)^{3/2}} \iiint e^{\mathrm{i}\boldsymbol{rk}} \langle \boldsymbol{HH}\rangle \mathrm{d}^3\boldsymbol{r}$$

$$\langle \boldsymbol{AH}\rangle(k) = \frac{1}{(2\pi)^{3/2}} \iiint e^{\mathrm{i}\boldsymbol{rk}} \langle \boldsymbol{AH}\rangle(r) \mathrm{d}^3\boldsymbol{r}$$

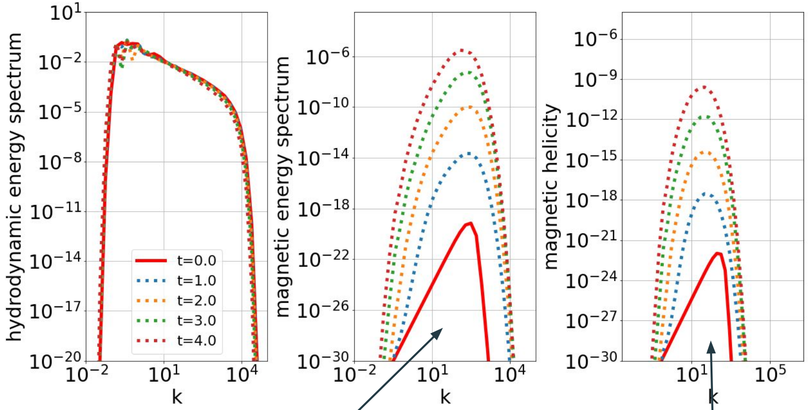
Thus, at sufficiently large magnetic Reynolds numbers on small scales, the magnetic helicity also grows exponentially. In this case, due to the law of conservation of helicity for three-dimensional ideal magnetohydrodynamics, helicity on small scales is divided by sign.

Small-scale helicity generation in shell models.

Kazantsev-type models are linear models that do not use the equation of motion and do not take into account the reverse effect of the magnetic field on the velocity field. They are not capable of describing any cascade of generated energy. However, shell models can [Ilyas' talk].

$$\boxed{\begin{aligned} d_t U_n &= ik_n(\Lambda_n(U,U) - \Lambda_n(B,B)) - \frac{k_n^2 U_n}{Re} \\ d_t B_n &= ik_n(\Lambda_n(U,B) - \Lambda_n(B,U)) - \frac{k_n^2 B_n}{Rm} \end{aligned}}$$

$$\begin{aligned} \Lambda_n(X,Y) = q^2(X_{n+1}Y_{n+1} + X^*_{n+1}Y^*_{n+1}) - X^r_{n-1}Y_n - \\ - X_n Y^r_{n-1} + iq(2X^*_n Y^i_{n-1} + X^r_{n+1}Y^i_{n+1} - X^i_{n+1}Y^r_{n+1}) + \\ + X_{n-1}Y_{n-1} + X^*_{n-1}Y^*_{n-1} - q^2(X^r_{n+1}Y_n + X_n Y^r_{n+1}) + \\ + iq(2X^*_n Y^i_{n+1} + X^r_{n-1}Y^i_{n-1} - X^i_{n-1}Y^r_{n-1}) \end{aligned}$$



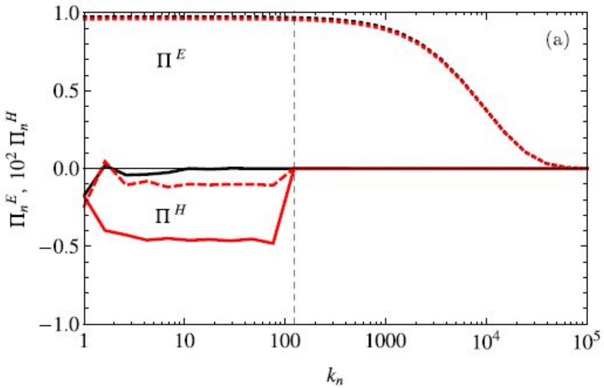
$$E_t = \sum (U_n U_n^* + B_n B_n^*)/2$$

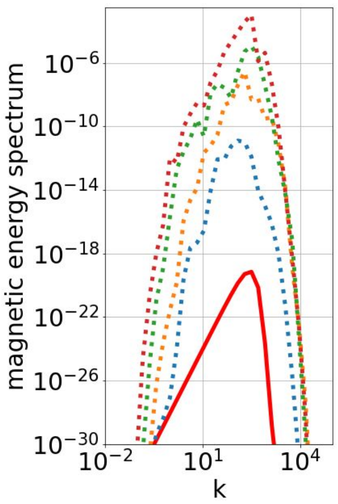
$$H_m = \sum i((B_n^*)^2 - (B_n)^2)/4k$$

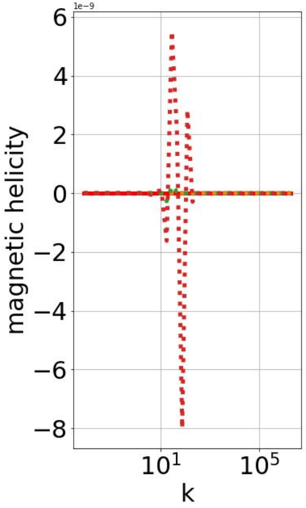
Problems of helicity transport.

[Stepanov et al., AJL, 2015]

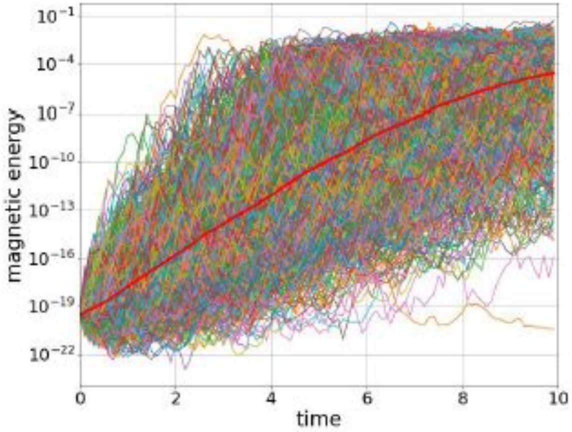
Cascade models are based on conservation laws through collective variables. They also demonstrate a separation of helicity in sign during the operation of a small-scale dynamo, which occurs simultaneously with exponential growth. However, this raises the problem of adequate averaging of the unstable numerical data.







The second problem is related to the fact that these models show a reverse cascade of helicity, but only constantly pumped up on small scales. A small-scale dynamo, in the process of amplifying the magnetic field in them, simply stops.



Main conclusions

A small-scale dynamo turns on in a short-correlated turbulent flow at sufficiently large magnetic Reynolds numbers. Along with magnetic energy, it generates magnetic helicity, which is separated by sign and grows exponentially in the kinetic approximation. It is possible to obtain the asymptotics of the corresponding threshold Rm values, correlation functions, and their spectra. However, the question remains whether there is a reverse cascade of this helicity from small to large scales.

Along with short-correlated linear models, magnetic helicity is also generated in nonlinear shell models for sufficiently large magnetic Reynolds numbers. In this case, when a balance is established between the fluid-dynamic and magnetic energies in the nonlinear model, small-scale generation stops. Moreover, to all appearances, the large-scale part of the hydrodynamic spectrum is responsible for small-scale generation, since an increase of the inertial interval leads to the generation suppression. The shell model describes the reverse helicity cascade at constant pumping, however, in the process of its development, small-scale generation stops and the reverse cascade also stops. Thus, the question of whether small-scale effects influence large-scale generation is still open (especially for experimental verification)!

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